A DISEQUILIBRIUM ECONOMETRIC ANALYSIS OF INTEREST RATE FUTURES MARKETS

BY

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A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL OF THE
UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

1981

ACKNOWLEDGMENTS

I wish to thank Professors William Howard, G. S. Maddala, and Arnold Heggestad for their interest and criticisms.

This study also benefited from many discussions with Professor Fred Arditti, who provided both encouragement and criticism when they were needed.

I am indebted to Alletta Jumper for typing this dissertation. Financial support during the course of this study from the Canada Council, the Social Sciences and Humanities Research Council of Canada, and the Center for Econometrics and Decision Sciences at the University of Florida is gratefully acknowledged.

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KEY TO SYMBOLS AND ABBREVIATIONS

$A_0(t)$	The average size of the asset base of speculators at time t.
$A_1(t)$	New mortgages at time t plus the increase in outstanding commitments to borrowers over the previous time period.
$A_2(t)$	The increase in forward sales of assets for the month ending at time t plus expected cash to be invested for the next quarter by financial institutions.
$b(t)$	The basis at time t, defined as the futures price minus the spot price.
C	The coupon payment for one hundred dollars face value of Government National Mortgage Association certificates, for the time the certificate is held by the hedger.
CAPM	Capital Asset Pricing Model.
CDR	Collateralized Depository Receipt.
CFTC	Commodity Futures Trading Commission.
D_t	The number of futures contracts demanded at time t.
E_i	Expected rate of return on asset i.
E_m	Expected rate of return on a portfolio of all assets in the market
GNMA	Government National Mortgage Association
h	Expected return from a hedged position.
OLS	Ordinary Least Squares.
$p(t)$	The futures price at time t.
Q_t	The number of futures contracts traded on day t.

r	The correlation coefficient between the futures price and the spot price.
$r(t)$	The rate at which one can borrow money using a GNMA certificate as collateral.
S_t	The number of futures contracts supplied at time t.
$s(t)$	The spot price at time t.
T	The time of maturity of the futures contract.
u	The expected return from the unhedged position.
x	The ratio of hedged to unhedged assets.
2SLS	Two Stage Least Squares.

Abstract of Dissertation Presented to the Graduate
Council of the University of Florida in Partial Fulfillment
of the Requirements for the Degree of Doctor of Philosophy

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March 1981

Chairman: William M. Howard
Major Department: Finance, Insurance and Real Estate

The objects of this study are to develop a model of the supply and demand for interest rate futures contracts, to test the model using conventional econometric techniques, and to identify the disequilibrium econometric techniques which should be used to measure supply and demand schedules for this market.

Futures contracts for financial securities (interest rate futures) have become prevalent only since 1975. While much has been written on the uses of interest rate futures, no attempt to model the determinants of supply and demand schedules for such contracts, or to measure empirically these supply and demand schedules is reported in the literature. However there has been some research in the construction of models of supply and demand of commodity futures, some of which is applicable to interest rate futures. Accordingly, a model for interest rate futures is developed from work more in 1979 by [illegible] for commodity futures, with modifications for the unique characteristics of interest rate futures. The net demand for contracts by hedgers is derived from the expression for the optimal ratio of hedged to unhedged assets, with the hedged and unhedged assets considered as a portfolio. The factors which affect this ratio are analyzed, leading to statements concerning the expected effect of an increase in each of the factors on the demand for interest rate futures. The net supply of contracts by speculators is derived from an expression for the optimal number of contracts to be bought or sold by a speculator who maximizes expected utility. Factors affecting this optimal number are examined, and statements are made concerning the expected effect on the supply of interest rate futures of an increase in these factors.

To test the resulting model, a sample is taken from the market for contracts for future delivery of Government National Mortgage Association certificates for the years 1976, 1977, and 1978. The estimates of supply and demand schedules are derived from the sample by ordinary least squares and two stage least squares. While the coefficients of the variables in the schedules calculated by ordinary least squares indicate that both the supply and demand schedules are positive functions of the price of futures contracts, the schedules based on two-stage least squares—more

appropriate for simultaneous equations--do not. These latter schedules indicate that supply increases and demand decreases as the price of the futures contracts rises, as predicted by the model. The coefficient of the variable for expected profit is significantly positive in both the supply and demand functions. The coefficient of the return on the unhedged position is significantly negative, as predicted. However, for the variables measuring assets to be hedged and the asset base of speculators, the supply and demand schedules do not appear to behave as predicted by the model. It is concluded that these variables do not provide a sensitive measure of the underlying factors.

The testing of a model for interest rate futures presents some unique econometric problems. These futures markets may sometimes be in disequilibrium, since trading on the Chicago Board of Trade is regulated by the Commodity Futures Trading Commission and daily price movements cannot exceed specified amounts. An econometric model of estimation for this unusual source of disequilibrium contains a two-limit tobit type of model to accommodate these daily upper and lower limits on price. Such a model will be useful for markets with a large proportion of disequilibrium points. However, for the sample studied, the technique will not provide estimates far different from the two-stage least squares estimates because of the small number of disequilibrium points. For this reason, the interpretations of Chapter IV are maintained.

CHAPTER I
INTRODUCTION

A futures contract for an asset is a contract to buy or sell that asset at a specified future date for a price agreed upon at the time of the contract. Unlike forward commitments, which are private agreements between two individuals about later sales of assets, a futures contract conforms to standardized specifications and is traded in an organized market. Specifications include the method and date of delivery as well as the nature of the item to be delivered. Futures contracts are traded through a clearing house, which acts as an intermediary. Buyers and sellers of these contracts hold agreements with the clearing house rather than directly with each other. Since each transaction is backed by the clearing house, the risk of default on contracts is minimized. The arrangement also eliminates the problem of having to find a second party willing to buy (or sell) the contract being sold (or bought).

While futures contracts for agricultural commodities have been traded for over a century, the first organized trading of futures contracts for financial securities—often referred to as interest rates futures—occurred in October, 1975. This study is confined to the market for interest rate futures, concentrating on the futures contract which has the certificates of the Government National Mortgage Association (GNMA) as the security to be delivered.*

Before selling GNMA certificates, a savings and loan company or a mortgage company will lend out mortgage money, assembling a pool of mortgages in the process. This pool must contain only mortgages guaranteed by the Federal Housing Authority or the Veterans Administration, all with identical coupon rates and thirty-year maturities. Once a pool is assembled with a minimum value of one million dollars, the firm may apply to the Government National Mortgage Association for approval to issue certificates representing shares in the mortgage pool. The firm originating the mortgages bears the responsibility for making the interest and principal payments to the buyer of the certificate. The payments are guaranteed by the Government National Mortgage Association, so the certificates are called GNMA pass-through certificates, and are often referred to as GNMA's or "Ginnie Mae." The minimum principal value of a certificate is $25,000. The price of the GNMA certificate, called the spot price, is quoted as a percentage of par. Price fluctuations are quoted in thirty-seconds of a point. Since the average prepayment period on thirty-year mortgages in pools underlying GNMA certificates is twelve years, a maturity of twelve years is assumed and simplifying the yield on GNMA certificates.

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*The GNMA futures contract actually calls for delivery of a Collateralized Depository Receipt (CDR) which reflects the fact that the originator of the contract has $100,000 minimum principal balance in GNMA certificates with 8 percent interest, or equivalent, in an approved depository.

The GNMA futures contracts (futures contracts to buy or sell GNMA certificates) considered in this paper are traded through the Chicago Board of Trade. The buyer agrees to take delivery of the certificate on the date specified by the contract. The price at which the contract is bought or sold--the futures price--is the price which the buyer must pay and the seller must accept for the certificate at the time of delivery. The only money exchanged at the time of the futures transaction is the margin money. Futures traders must hold a minimum amount of money, called margin, in their accounts with their brokers. As the futures price changes from day to day, traders make or lose money, which is added to or taken away from their accounts at the end of each day. The margin money acts as a "cushion" from which the broker deducts any losses the trader might have incurred during the day.

Futures contracts in GNMA's may be used by hedgers in attempts to offset possible losses in their positions of GNMA certificates which result from changes in the spot price. Examples of such hedges are given in Exhibits 1 and 2 of Chapter III. Speculators also trade interest rate futures. Having no position in the underlying asset, they buy and sell futures contracts in order to profit from the movement in the futures price. Hedgers and speculators form the market for GNMA futures and it is by studying the actions of these two groups that we can define the factors that determine the supply and demand of GNMA futures contracts.

Much has been written about the purposes and efficiency of commodity futures markets and, more recently, interest rate futures markets, and most empirical work in futures markets has concentrated on price movements and market efficiency. However, there is some disagreement as to the primary nature of hedging, and how much hedging should be done in these markets by any firm. Furthermore, there have been relatively few efforts to create models describing how supply and demand are determined for any type of futures contract. There has been little attempt to measure empirically the factors that determine the supply and demand of any futures contracts.

Any attempt to measure supply and demand functions in futures markets is complicated by the fact that price movements of futures contracts are regulated by the Commodity Futures Trading Commission. Over the period from 1976 through 1978, the price of a GNMA futures contract was not allowed to vary by more than twenty-four thirty-seconds of a point above or below the previous day's closing price. If the price change reached this limit for three days in a row in all contracts specifying different months of delivery, a "variable price limit" of thirty-six thirty-seconds went into effect for the fourth and subsequent days. Whenever such price limits are in effect, the price mechanism no longer operates to allow markets to clear, causing disequilibrium.

For example, when the regulation-limited price is lower than the market-clearing price, there will be demand in excess of the quantity of contracts traded at that price.

The fact that futures markets may sometimes be in disequilibrium means that standard regression techniques cannot be used in the estimation of supply and demand schedules. Whenever a disequilibrium occurs, the distribution of the quantity variable is a truncated normal distribution, violating an important assumption of standard regression techniques.

The purpose of this study is threefold. First, an appropriate model of supply and demand for interest rate futures contracts is developed from Stein's (1979) analysis. Second, the correct estimation techniques for a model with this unusual source of disequilibrium is then developed. Third, five hypotheses concerning the signs of the coefficients of the supply and demand schedules are tested for the GNMA futures market using standard methods of estimation, and the effect of the correct techniques on the results is discussed.

Chapter II is a review of previous research in the areas of futures markets and estimation techniques for markets which may be in disequilibrium. An appropriate model of supply and demand for interest rate futures contracts is developed in Chapter III. The preliminary testing of the hypotheses is described in Chapter IV, which includes a description of the sample used, definitions of the variables, and preliminary test results, which would be appropriate as estimates in the absence of regulation. Chapter V is a description of the econometric techniques which are developed to allow for the possibility of regulation-induced disequilibrium in the GNMA futures market. The effect on the estimates for the model of using the correct estimation techniques is discussed. Finally, Chapter VI is a summary and a statement of the conclusions of the study.

CHAPTER II
PREVIOUS RESEARCH IN FUTURES MARKETS AND DISEQUILIBRIUM TECHNIQUES

This chapter is a review of research conducted in the areas of futures hedging and speculation, and of econometric techniques for markets which are sometimes in disequilibrium.

Futures Markets

Many examples can be found of the uses of futures contracts for both commodities and financial securities [See for example, Baron and Williams (1976), Lubchenco and Wagner (1976), and Sandor (1975)]. Futures contracts can be used for hedging and speculation. Firms or individuals with a futures position in a security, but with no plans for using the security itself, may be considered to be speculators. On the other hand, a firm or individual that makes use of a financial security and takes a futures position in it, or in securities closely related to it, may be defined as hedgers. Hedgers use the futures contract as a temporary substitute for a subsequent financial transaction.* For example, a hedger in interest rate futures may have a requirement to sell a security at a later date. Since a decline in the price of the security before the sale date will result in a loss, the hedger sells futures contracts in the security to protect against such a loss. If futures prices move with spot prices, the decline in the futures price would result in a gain on the short position, partially or completely offsetting the loss on the security.

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*This definition of hedgers is the one most commonly used in the industry and by the CFTC. By this definition

Early research contained the assumption that firms would attempt to eliminate all of the risk of price fluctuations by taking a futures position which completely offsets the position in the security or asset. Such a hedge is called a complete hedge. A complete hedge is called a perfect hedge--one in which the attempt to eliminate the risk of price fluctuations is successful--if the futures price is perfectly positively correlated with the spot price.

Working (1953), (1962) gives two reasons for firms to hedge. The first is to take advantage of relative price changes. If the basis, defined as the futures price minus the spot price, is expected to decrease, the firm will sell futures to offset inventory in anticipation of a gain on the total position when the futures price falls by more than the spot price. The second reason is to protect against

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firms or individuals who have plans to sell securities at a later date are only hedgers if they sell futures contracts for those securities, or ones closely related to them.

declines in the spot price when the firm is holding inventory. Working claims that the firm will not hedge unless one of these occurrences is anticipated. Thus, he states that firms are not often complete hedgers, and that they hedge to maximize expected return rather than to minimize risk. Johnson (1960) defines a hedge as a position in two markets such that the price risk of holding given quantities in both markets over a specified time is minimized. He introduces the concept of evaluating the expected return as well as the price risk before determining the optimum position to take in the futures market. In Johnson's framework, a firm will hedge to minimize the price risk for a given expected return on the total position of inventory and futures contracts, or maximize expected return for a given level of price risk. The optimal number of futures contracts to buy or sell is therefore determined by the desired expected return as well as the reduction in price risk that a futures position might provide. A complete hedge is not always optimal in Johnson's framework, and we see that Working's examples are consistent with Johnson's model. Working described firms which maximize expected return, while Johnson's model contains the objective of maximizing expected return for a given level of price risk. Working states that firms are not complete hedgers because they do not attempt to minimize price risk, but Johnson illustrates how a complete hedge may not be optimal for price risk minimizers as well, when expected return on the total position is part of the objective.

While futures markets allow for the transfer of price risk from one party to another, there have been differing views as to the exact nature of these markets. The Keynesian view (1930) is that futures markets provide the opportunity for speculators to sell insurance to hedgers against price fluctuations. As in conventional securities markets, the speculators should expect to receive a risk premium for bearing the price risk. Hardy's (1940) opposing view is that futures markets are closest in nature to gambling casinos, where speculators actually pay for the privilege of gambling on price movements. This theory implies that a negative risk premium should exist in futures markets, making them completely different from conventional markets.

Dusak's study (1973) examines the futures markets in wheat, corn and soybeans in the context of the capital asset pricing model (CAPM). First developed by Markowitz (1959), Lintner (1965), and others, the CAPM provides a linear relationship between the equilibrium expected return and the risk of any asset under certain assumptions* when investors are risk-averse. The linear relation is written as

$$E_i = r + \beta_i [E_m - r] \qquad (1)$$

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*The assumptions are: 1) Individuals may borrow or lend as much as desired at a riskless rate of return, r, 2) individuals have homogeneous expectations, and 3) all information about the attractiveness of any investment is contained in its mean and standard deviation where

E_i = the expected one-period rate of return on asset i;

r = The one-period rate of return on the riskless asset.

E_m = the expected one-period rate of return on the "market portfolio," which is the portfolio containing all assets on the market.

β_i = the covariance of the return on asset i and the return on the market portfolio, divided by the variance of the market return, written as $\sigma(R_i, R_m) / \sigma^2 (R_m)$.

According to the CAPM, the risk of the asset i is given by β_i. Equation (1) can be rewritten as

$$E_i - r = \beta_i [E_m - r] \qquad (2)$$

The left hand side of equation (2) may be viewed as a risk premium on asset i--the excess of the expected return above the riskless rate--and the right hand side shows how the risk premium is affected by the asset's risk, β_i. In applying the CAPM to wheat, corn and soybean futures, Dusak finds both β's and risk premiums in all three to be approximately equal to zero. These findings do not contradict the theories of either Keynes or Hardy.

There is, however, neither evidence nor reason to suspect that futures markets are different in nature from other markets. Unless there are barriers to entry into futures markets, the equilibrium risk return trade-off should be the same there as in cash markets. Using the assumption that the equilibrium risk-return relationship is the same for futures markets as for cash markets, Black (1976) derives an equilibrium expected futures price based on the CAPM. The expected change in the futures price from time t to (t+1) is given by

$$E[\,p(t+1) - p(t)] = \beta^*[R_m - r] \tag{3}$$

where

$p(t+1)$ = futures price at time (t+1), a random variable.

$E(\cdot)$ = expected value of the expression in parentheses.

$$\beta^* = \frac{\sigma[p(t+1) - p(t), R_m]}{\sigma^2(R_m)}$$, a "dollar β"

which has as the numerator the covariance of change in the futures price and the return on the market portfolio, rather than the usual $\sigma(R_i, R_m)$, as in equation (2).

Equation (3) provides the expected increase in the futures price which would be sufficient compensation for the risk of the futures contract (its β^*) if futures markets are similar in nature to other markets; that is, if investors in futures markets are assumed to be risk-averse, as they are in other markets. Let us suppose for a minute that [illegible] theory is correct, and that futures speculators are actually willing to pay for the privilege of gambling in the market. Then a speculator would buy a futures contract even if the expected price appreciation was less than the amount given by equation (3). Thus, a futures contract having an expected price appreciation given by

$$E[p(t+1) - p(t)] < \beta^*[E_m - r] \qquad (4)$$

is acceptable to such speculators (even though they could take on the same risk at a higher expected return by investing in other markets, where equation (3) should hold).

However, risk-averse investors--hedgers--form the other side of the market. If speculators, accepting price movements characterized by equation (4), provide the net demand for futures contracts, then the risk-averse hedgers provide the net supply of futures contracts. Hedgers sell futures contracts to reduce the price risk of their position. The expected cost from time t to (t+1) of this reduction in risk is given by $E[p(t+1) - p(t)]$. Now, according to equation (3) which is derived from the CAPM, any risk-averse investor would be willing to pay up to the expected amount of $\beta^*[E_m - r]$ for the reduction in risk given by the β^* of the contracts sold. Before entering into a futures transaction, potential hedgers will consider other markets--which provide the same opportunity for risk reduction given by the future market--to see which market provides this reduction at the lowest expected cost.

Similarly, potential investors in other markets wishing to reduce risk in their portfolios may consider the sale of futures contracts as one way to achieve that goal. In any market dominated by risk averse investors, the equilibrium expected cost of reducing portfolio risk by the amount β^* will be $\beta^*[E_m - r]$. As long as equation (4) holds in the futures market, the expected cost of reducing risk in that market, $E[p(t+1) - p(t)]$, is lower than the corresponding cost in any other market. After comparing costs, hedgers and other investors wishing to reduce portfolio risk would choose to sell futures contracts rather than using other available markets at a higher expected cost, increasing the supply of futures. As the supply increases, the price, $p(t)$, will decrease, thereby increasing the expected cost, $E[p(t+1) - p(t)]$.

As long as the futures market is small relative to all securities markets, investors will continue to enter the futures market and drive down the futures price until equation (3) holds in that market, even though the speculators are willing to accept equation (4). Therefore, Hardy's thesis on a negative risk premium in futures markets cannot hold unless there are forces which prevent entry into futures markets, or futures markets are very large relative to all other markets, the same equilibrium risk-return relationship should exist in futures markets as in other markets.

It is only after rejecting Hardy's viewpoint on the nature of futures markets that the pricing of futures contracts by the methods associated with the CAPM becomes classical, and the research on hedging that followed the CAPM does make use of these methods. We recall that Johnson was one of the first to assume an objective of minimizing price risk for a given level of expected return to determine the optimal number of futures contracts for a hedge. Stein (1979) uses the same concept, further developed in the CAPM itself, to derive an expression for the net supply of commodity futures contracts by hedgers. The optimal quantity of hedged stock for a firm is derived as a function of several variables, and an aggregate supply schedule is developed from the functions of the individual firms. Stein then develops an expression for the net demand for commodity futures contracts, which is based on the assumption of risk-averse behavior on the part of speculators in the market. Stein supports the assumption on the grounds that large speculating firms would have little success in attracting capital if they did not offer the same risk-return relationship to investors as other firms did. A general utility function for speculators is then used to derive the aggregate excess demand schedule.

The derivation of functions for demand and supply of interest rate futures contained in Chapter III of this study is based primarily on the work of Stein, with adaptations to accommodate the differences between interest rate futures and commodity futures.

Econometric Techniques for Markets Which May Be in Disequilibrium

If futures markets were allowed to function freely, we could write the expressions for the demand and supply of a future contract as a system of simultaneous equations such as

$$D_t = \alpha_1 p(t) + \beta_1' X_{1t} + u_t \tag{5}$$

$$S_t = \alpha_2 p(t) + \beta_2' X_{2t} + v_t \tag{6}$$

and

$$Q_t = D_t = S_t, \tag{7}$$

where D_t is the number of futures contracts demanded on day t at price p(t), S_t is the number of futures contracts supplied on day t at price p(t), and Q_t is the number of futures contracts traded on day t. X_{1t} and X_{2t} are K x 1 vectors of exogenous variables, α_1 and α_2 are scalars, and β_1 and β_2 are K x 1 vectors. If the market is in equilibrium, the price, p(t), will be that for which supply and demand are equal and equation (7) holds, and standard simultaneous equation techniques such as two stage least squares (2SLS) may be used to estimate the supply and demand functions. However, the regulation of the futures price by the Commodity Futures Trading Commission creates the need for a different estimation technique.

Futures markets may be in disequilibrium whenever price limits are met. Instead of a model where the quantity of futures contracts traded is the market-clearing quantity, when a price limit is met, the quantity transacted will equal the minimum of the supply or the demand at that price. For example, if a price decrease is stopped when the lower price limit is met, the regulation is holding the price artificially high and the supply of futures contracts will exceed demand at that price. The quantity of futures contracts traded is then the quantity demanded. Similarly, when a price increase is stopped by regulation as the upper limit is met, the quantity traded is the quantity supplied at the limit price.

Whenever the price limits are in effect, price is no longer an endogenous variable in the model; $p(t) = p(t-1) \pm L$, where L is the regulated maximum price movement Here, $Q_t = D_t$ when $p(t) = p(t-1) - L$ and $Q_t = S_t$ when $p(t) = p(t-1) + L$. If $p^e(t)$ is the market-clearing price, $\Delta p = p(t) - p(t-1)$, and L is the regulated maximum price movement, then

$$Q_t = D_t = S_t \text{ when } |\Delta p| < L$$

and

$$Q_t = \begin{cases} S_t , & \Delta p = +L \\ D_t , & \Delta p = -L \end{cases}$$

So the model is then

$$D_t = \alpha_1 p(t) + \beta_1' X_{1t} + u_t \tag{5}$$

$$S_t = \alpha_2 p(t) + \beta_2' X_{2t} + v_t \tag{6}$$

$$Q_t = \min (D_t, S_t) \tag{7}$$

$$|\Delta p| = \min (|p^e(t) - p(t-1)|, L) \tag{8}$$

Observations can be separated into equilibrium observations and those from the demand or the supply side of the market.

Even though observations are identified as those from the demand or the supply function, the method of ordinary least squares (OLS) will not give consistent or unbiased estimates for those observations. The residuals from the truncated samples which we observe do not have zero means and they are correlated with the exogenous variables.

Tobin (1958) was one of the first to consider the problem of estimating a model with a limited dependent variable. The resulting "tobit" method combined probit analysis and multiple regression techniques, and specified the correct likelihood function for models where there are concentrations of the dependent variable at a limit. Olsen (1978) shows that tobit likelihood functions have a unique global maximum.

Fair and Jaffee (1972) considered four types of disequilibrium models using OLS and 2SLS methods, and conducted some of the first empirical work in this area. Fair and Kelejian (1974) expanded on the work of Fair and Jaffee by providing a maximum likelihood estimator for one of the models outlined.

In later papers, Amemiya (1974) and Maddala and Nelson (1974) gave the correct unconditional likelihood function and statistical techniques for multivariate and simultaneous equation models, including disequilibrium models. Amemiya (1973) showed that the 2SLS estimator is consistent but not asymptotically efficient, while the new variant proposed by Fair and Jaffee is not consistent.

All of the techniques mentioned above were developed for models in which the dependent variable has either an upper or lower limit. In futures markets, the price has both an upper limit and a lower limit, requiring an estimation technique which is somewhat different from those above. For the reduced form of the supply and demand equations, when the price is regressed against the exogenous variables in the model, the likelihood function must reflect both the upper and lower limits. Therefore, while all of the preceding work dealt with models similar to the one being studied, none of the likelihood functions discussed is appropriate for the model being studied.

The first work done for models in which the dependent variable has both upper and lower limits was by Rossett and Nelson (1975), who defined the correct likelihood function to be used in probit analysis. Fishe and Lahiri (1981) make use of a likelihood function which reflects both upper and lower limits, and use the function in estimating expectations from survey data.

The likelihood function necessary to reflect the price limits in futures markets is described in detail in Chapter V.

CHAPTER III
A MODEL OF THE DEMAND AND SUPPLY OF INTEREST RATE FUTURES CONTRACTS

A model of supply of and demand for interest rate futures contracts is developed in this chapter. The model is based on concepts used in Stein's work on commodity futures contracts (1979), with adaptations and extensions to accommodate the specific characteristics of interest rate futures contracts. A firm's uses of interest rate futures are first discussed, and returns and standard deviations from hedged and unhedged positions, both long and short, are derived. The optimal ratio of hedged to unhedged contracts is then considered as a portfolio problem of minimizing the standard deviation of the return of the portfolio for a given expected return for each position. Next, the net demand for futures is calculated as the demand less the supply by hedgers. Finally, the net speculative supply of futures is determined, yielding a complete model of supply of and demand for interest rate futures.

This model is applied to futures contracts for GNMA certificates.

Hedgers

We first consider the uses of interest rate futures for a hedging firm. A firm may require a short hedge or a long hedge, depending on its financial requirements.

A short hedge involves the sale of futures contracts and gives some protection against a rise in interest rates. Firms with assets (or commitments to buy such assets at a fixed price) which decline with rises in interest rates, and firms with future requirements to borrow funds are those most likely to use a short hedge. For example, a savings and loan association may have a commitment to buy mortgages at the current GNMA yield. Not being able to sell a complete pool until a later date, they wish to hedge against interest rate rises which would decrease the value of the pool. Exhibit 1 shows a short hedge which would protect against interest rate rises.

A long hedge involves the purchase of futures contracts and provides some protection against interest rate decreases. A firm wishing to lock in a lending rate may attempt to do so through a long hedge. A savings and loan association with a commitment to sell mortgages at a fixed price, or with an anticipation of funds to invest, may want to be able to invest the funds later at rates close to current ones. A long hedge helps in the effort to lock in the current rate. Exhibit 2 illustrates such a long hedge.

We must note that both of these exhibits are examples in which the basis, defined as the futures price minus the spot price, was constant over the duration of the position, enabling the hedger to lock in the current rate in both cases. Since both prices moved by the same amount, the loss in the

EXHIBIT 1
EXAMPLE OF A SHORT HEDGE

	Cash	Futures
January	Savings and Loan commits $1 million at the current GNMA yield of 8.5%. Price = 96 3/32 = $960,937.50	Company sells 10 GNMA contracts for delivery in June. Price = 95 3/32 = $950,937.50
April	Company completes pool and sells $1 million of GNMA 8's at a yield of 9%. Price = 92 22/32 = $926,875.00	Company closes its futures position by buying 10 GNMA contracts for delivery in June. Price = 91 22/32 = $916,875.00
	Loss = $34,062.50	Gain = $34,062.50

EXHIBIT 2
EXAMPLE OF A LONG HEDGE

	Cash	Futures
June	Savings and Loan plans to buy GNMA's in September and wants to lock in the current rate of 8.514%. Price = 96 = $960,000.00	Savings and Loan buys 10 GNMA contracts for delivery in September. Price = 95 14/32 = $954,375.00
August	The yield falls to 8% and the company buys in GNMA's. Price = 99 21/32 = $996,562.50	Company closes its futures position by selling 10 GNMA contracts for September delivery. Price = 99 3/32 = $990,937.50
	Loss = $36,562.50	Gain = $36,562.50

spot position was exactly offset by the gain in the futures position. There is, however, no guarantee that the basis will remain constant. For example, as the time to maturity of the futures contract decreases, the futures price should approach the spot price, since the futures price must equal the spot price at the maturity date. When the futures price is below the spot price, as in our exhibits, the basis would increase (become less negative) over time and the long hedger would gain while the short hedger would lose.

We can now specify the returns for the long and short hedger, summarized in Tables 3-1 and 3-2. We shall use the following notation:

$p(t)$ = price at time t of futures contract maturing at time T,

$s(t)$ = spot price of security at time t

$b(t)$ = basis = $p(t) - s(t)$

x = ratio of hedged to unhedged securities

τ = a point in time between t and T

C = coupon payment from time t to τ per security.

$r(t)$ = repurchase rate per time t to τ, which is the rate at which the firm could borrow money to invest in securities

h = expected return from the hedged position

u = expected return from the unhedged position

r = correlation coefficient between $p(t)$ and $s(t)$

ρ = correlation coefficient between the hedged and unhedged returns.

TABLE 3-1
RETURNS FROM LONG AND SHORT HEDGES

A. The Short Hedge

A security dealer may borrow money at the rate of $r(t)$ for the time t to τ, and use it to buy a financial security priced at $s(t)$. He simultaneously sells $x/(1 + x)$* futures contracts, maturing at time T, where $t < \tau < T$, at a price of $p(t)$. At time τ, he sells the securities, now at a price of $s(\tau)$. He also cancels his futures position by buying $x/(1 + x)$ futures contracts, priced at $p(\tau)$. In addition to any gain or loss on the transaction he has earned \$C from coupon payments on the asset. His payments are $s(t)[1 + r(t)]$ and $p(\tau) \cdot x/(1 + x)$, and his receipts are $s(\tau)$, C, and $p(t) \cdot x/(1 + x)$. His total return per \$s(t) is therefore,

$$s(\tau) - s(t) + [p(t) - p(\tau)] \cdot x/(1 + x) \\ + [C - r(t) \cdot s(t)]$$

B. The Long Hedge

A security dealer expects a cash inflow to invest in financial securities at time τ, but would like to lock in the current rate, which he considers favorable. To do this, he must either buy the asset at time t, borrowing the funds to do so, or use the futures markets. He decides to do this without any outflow at time t by using the futures market. Since he is not investing, he foregoes the coupon income of C from time t to τ. At the same time, however, he does not have to pay any interest on the loan he would have had to take to buy the asset at time t. So he is also foregoing interest payments of $r(t) \cdot s(t)$ by not investing at t. Instead, he buys $x/(1 + x)$ futures contracts at price $p(t)$. At time τ, when he receives the money to invest, he buys the security, now priced at $s(\tau)$. He also cancels his futures position by selling $x/(1 + x)$ futures contracts at the new price, $p(\tau)$. The total payments are $p(t) \cdot x/(1 + x)$, $s(\tau)$, and the foregone receipt of C, and the total receipts are $p(\tau) \cdot x/(1 + x)$ and the foregone payments of $s(t)[1 + r(t)]$. Thus, his total return per \$s(t) "received" is:

$$s(t) - s(\tau) + [p(\tau) - p(t)] \cdot x/(1 + x) \\ - [C - r(t) \cdot s(t)]$$

---

*If x is the ratio of hedged to unhedged assets, then $x/(1 + x)$ is the fraction of assets which are hedged.

TABLE 3-2
RETURNS AND VARIANCES FROM LONG AND SHORT POSITIONS

	Hedged*	Unhedged
Short		
Return (Total)	$= s(\tau) - p(\tau) - s(t) + p(t) + [C - r(t) \cdot s(t)]$ $= b(t) - b(\tau) + [C - r(t) \cdot s(t)]$	$= s(\tau) - s(t) + [C - r(t) \cdot s(t)]$
Expected Return	$= b(t) - E[b(\tau)] + [C - r(t) \cdot s(t)]$ $= h$	$= E[s(\tau)] - s(t) + [C - r(t) \cdot s(t)]$ $= u$
Variances	$= \sigma^2_b = \sigma^2_s + \sigma^2_p - 2r\,\sigma_s\sigma_p$ $(= \sigma^2_h)$	$= \sigma^2_s$ $(= \sigma^2_u)$
Long		
Return (Total)	$= s(t) - p(t) - s(\tau) + p(\tau) - [C - r(t) \cdot s(t)]$ $= b(\tau) - b(t) - [C - r(t) \cdot s(t)]$	$= s(t) - s(\tau) - [C - r(t) \cdot s(t)]$
Expected Return	$= E[b(\tau)] - b(t) - [C - r(t) \cdot s(t)]$ $= h_1$	$= s(t) - E[s(\tau)] - [C - r(t) \cdot s(t)]$ $= u_1$
Variances	= same as for the short	= same as for the long

---

*For a completely hedged position, $x/(1 + x) - 1$.

The variables p(t), s(t), and b(t) are random variables until time t. All of the following analysis is for a futures contract maturing at time T, where $t < \tau < T$.

From Table 3-1, we can write a general expression for the net return from time t to τ per \$s(t) of the asset, where $x/(1 + x)$ of the assets (or commitments) are hedged, as

$$s(\tau) - s(t) + [p(t) - p(\tau)] \cdot x/(1 + x) \\ + [C - r(t) \cdot s(t)] \qquad (1)$$

for a short hedge, and as

$$s(t) - s(\tau) + [p(\tau) - p(t)] \cdot x/(1 + x) \\ - [C - r(t) \cdot s(t)] \qquad (2)$$

for a long hedge.

Having determined the expected returns and standard deviations for the hedged and unhedged positions in Tables 3-1 and 3-2, we may now consider the optimal ratio, x, for the long or short hedger as a portfolio decision. We will not discuss here the manner in which the firm determines the total amount of securities to hold (or sell). The expected returns have been written as returns per s(t) dollars of investment (or sale) on the assumption of large, well functioning markets in which the firm cannot affect the futures or spot price. The firm can therefore determine its optimal ratio of hedged to unhedged assets independently of its operating decisions. It can then use this information in forming expectations of the price of its assets (or liabilities) for its operating decisions.

We first consider the optimal ratio x for the short hedger. Let E_h and E_u be the fractions of the total investment placed in hedged and unhedged positions, respectively. Then

$$x = E_h/E_u \tag{3}$$

In the analysis which follows, we standardize s(t) at $1.00 in order to define all returns as net rate of return per dollar. Alternatively, we could have adjusted by using h/s, u/s, and σ_h/s and σ_u/s , obtaining the same results in the analysis.

We consider a portfolio containing the hedged and unhedged positions, plus a risk-free asset with a rate of return of i percent. A portfolio of the risk-free asset and the two positions has an expected rate of return, E_p, of

$$E_p = E_h \cdot h + E_u \cdot u + (1 - E_h - E_u) \cdot i$$

or

$$E_p = i + E_h(h - i) + E_u(u - i) \tag{4}$$

and, letting σ_p denote the portfolio variance (noting that the capital letter subscript distinguishes it from the variance of the futures price),

$$\sigma_P^2 = E_h^2\sigma_h^2 + E_u^2\sigma_u^2 + 2E_hE_u\sigma_{uh} \qquad (5)$$

The firm attempts to select a portfolio with the minimum standard deviation for a given expected return. The problem may be stated as

$$\min_{E_hE_u} \sigma_P = (E_h^2\sigma_h^2 + E_u^2\sigma_u^2 + 2E_hE_u\sigma_{uh})^{1/2}$$

subject to

$$E_P = i + E_h(h - i) + E_u(u - i) \qquad (6)$$

The Lagrangian is therefore

$$\begin{aligned} L = \sigma_P - \lambda[(E_P - i) &+ E_h(h - i) \\ &+ E_u(u - i)]. \end{aligned} \qquad (7)$$

At the optimum fraction, E_h, the following conditions hold:

$$\frac{\delta\sigma_P}{\delta E_h} = \lambda(h - i), \qquad \frac{\delta\sigma_P}{\delta E_u} = \lambda(u - i)$$

or

$$\tfrac{1}{2}\ (2E_h \sigma_h^2 + 2E_u \sigma_{uh}) \sigma_p^{-1/2} = \lambda (h - i) \qquad (8)$$

and

$$\tfrac{1}{2}\ (2E_u \sigma_u^2 + 2E_h \sigma_{uh}) \sigma_p^{-1/2} = \lambda (u - i) \qquad (9)$$

Dividing equation (8) by (9) results in a ratio of excess hedged to excess unhedged returns, given by

$$\frac{(h - i)}{(u - i)} = \frac{E_h \sigma_h^2 + E_u \sigma_{uh}}{E_u \sigma_u^2 + E_h \sigma_{uh}} \qquad (10)$$

But, since we have defined x by the expression

$$x = E_h / E_u$$

then, we can rewrite equation (10) as

$$\frac{(h - i)}{(u - i)} = \frac{x\sigma_h^2 + \sigma_{uh}}{\sigma_u^2 + x\sigma_{uh}}$$

$$= (x\sigma_h + \rho\sigma_u)\sigma_h / (\sigma_u + x\rho\sigma_h)\sigma_u \qquad (11)$$

Multiplying both sides of (11) by σ_u / σ_h yields the following ratio of excess returns at the optimum

$$\frac{(h - i)/\sigma_h}{(u - i)/\sigma_u} = \frac{(x\sigma_h + \rho\sigma_u)}{(\sigma_u + x\rho\sigma_h)} \qquad (12)$$

Let

$$y = \frac{(h - i)/\sigma_h}{(u - i)/\sigma_u} \qquad (13)$$

The ratio of excess hedged return per unit of its standard deviation to excess unhedged return per unit of its standard deviation at the optimum, y, is given by (12). Solving for x from (12) and using y, given in (13), we find the optimal ratio of short hedged to unhedged assets, x*, for a given expected return;

$$x^* = \frac{(y - \rho)\ \sigma_u}{(1 - y\rho)\sigma_h} \tag{14}$$

This x* is the ratio of S_h/S_u all along the line iMB in Figure 1.

Now, the correlation coefficient between the hedged and unhedged returns, ρ, is defined as

$$\rho = \frac{\sigma_{uh}}{\sigma_u \sigma_h} \tag{15}$$

But, using the information for Table 3-2, σ_{uh} can be written as

$$\begin{aligned} \sigma_{uh} = {} & E\{ b(t) - b(\tau) - E[b(t) - b(\tau)]\}\{ s(\tau) - s(t) \\ & - E[s(\tau) - s(t)] \} \\ = {} & \sigma_{bs} \end{aligned} \tag{16}$$

The covariance between the hedged and unhedged returns is equal to the covariance between the spot price and the basis.

If the futures price moves exactly as the spot price then the basis would be a constant, the covariance between the basis and the spot price would be equal to zero, and the correlation coefficient, ρ, would also equal zero. Under the assumption that $\rho = \rho^*$, (14) becomes

$$x^* = \frac{y\sigma_u}{\sigma_h} = (h - i)/(u - i)(\sigma_u^2/\sigma_h^2) \qquad (17)$$

Referring to (16), σ_{uh} can also be written as

$$\sigma_{uh} = \sigma_s^2 - \sigma_{ps}$$

or

$$\sigma_{uh} = \sigma_s^2 - r\sigma_p\sigma_s \qquad (18)$$

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*Based on the sample years of 1976 to 1978, the covariance between the basis and the spot was close to zero for each contract studied. These statistics are contained in Table 4-3.

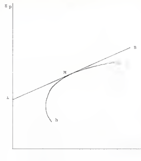

FIGURE 1
PORTFOLIOS OF HEDGED TO UNHEDGED SECURITIES

Here, u represents the unhedged position of inventories of financial securities alone, and h represents the hedged position of financial securities plus short sales of futures contracts. The curve hMu is the feasible set of portfolios in which the ratio varies from $x/(1 + x) = 1$ (at h) to $x/(1 + x) = 0$ (at u).

If ρ is approximately equal to zero, then σ_{uh} is approximately zero, and (18) can be written, using the approximation, as

$$\sigma^2_s = r\sigma_p\sigma_s \tag{19}$$

or

$$\sigma_s = r\sigma_p$$

From Table 3-2

$$\sigma^2_h = \sigma^2_s + \sigma^2_p - 2r\sigma_p\sigma_s \tag{20}$$

and

$$\sigma^2_u = \sigma^2_s \tag{21}$$

Now, substituting (19) into (20) and (21) gives

$$\sigma^2_h = \sigma^2_p\ (1 - r^2) \tag{22}$$

and

$$\sigma^2_u = r^2\sigma^2_p \tag{23}$$

substituting (22) and (23) into (17), we get

$$x^* = [(h - i)/(u - i)(r^2/(1 - r^2))] \tag{24}$$

From (24), we can see that the ratio of hedged to unhedged assets for the short hedger is positively related to the ratio of the excess hedged to excess unhedged returns, $(h - i)/(u - i)$.

For a given optimal ratio of hedged to unhedged assets, x^*, the short hedger will decrease the number of futures sold with decreases in mortgages held or forward commitments to buy such mortgages. In addition, a decrease in the value of

a' will mean a decrease in the supply of futures by hedgers. The ratio of $(h - i)/(u - i)$ decreases with a decrease in the current basis or an increase in the expected basis. The ratio also decreases with decreases in $[C - r(t) \cdot s(t)]$.* Therefore, we can summarize by saying that short hedgers will decrease their aggregate supply of futures contracts with a) an increase in $E[b(\tau)] - b(t)$; b) a decrease in the difference between the GNMA coupon payment and the cost of the funds involved, $[C - r(t) \cdot s(t)]$; and c) a decrease in the aggregate amount of financial securities held and forward commitments to purchase such securities.

A similar analysis for the long hedger yields the same result as equation (24), with

$$h = E[b(\tau)] - b(t) - [C - r(t) \cdot s(t)] \qquad (25)$$

and

$$u = a(t) - E[a(\tau)] - [C - r(t) \cdot s(t)] \qquad (26)$$

Long hedgers will increase their aggregate demand for futures with increases in the aggregate amount of forward sales of mortgages, and increases in expected inflows of cash to be invested, for a given optimal ratio, x^*. If the optimal ratio of hedged to unhedged assets increases, the aggregate demand for futures by hedgers increases. As with short hedgers, x^* for long hedgers is positively related to the ratio of $(h - 1)/(u - 1)$. Factors which increase this ratio include an increase in the expected basis, a decrease in the current basis, and a decrease in $[C - r(t) \cdot s(t)]$.* In summary long hedgers will increase their aggregate demand for futures contracts with a) an increase in $E[b(\tau)] - b(t)$; b) a decrease in $[C - r(t) \cdot s(t)]$; and c) an increase in the aggregate amount of forward sales of financial securities and expected inflows of cash. We see that changes in basis and $[C - r(t) \cdot s(t)]$ which increase aggregate demand also decrease aggregate supply. Using the foregoing analysis, we can aggregate over all hedgers to formulate a function for the net demand for futures by hedgers, in which the hypotheses concerning the effect of a change in each factor are outlined in parts a), b) and c) of the two preceding paragraphs. These hypotheses are restated in terms of the coefficients of the variables at the end of the chapter.

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* $\frac{\delta[(h - i)/(u - i)]}{\delta[C - r(t) \cdot s(t)]} = \frac{E[p(\tau)] - p(t)}{(u - i)^2} > 0$:

On average, $E[p(\tau)] > p(t)$, since the futures price is typically below the spot price by a difference of about $[C - r(t) \cdot s(t)]$ in equilibrium (otherwise, arbitrage opportunities will occur). Since the futures price would approach the spot price as its maturity approaches, it should therefore be increasing over time, and $E[p(\tau)] > p(t)$. This is discussed further in Chapter IV.

$$D_t = \beta_{11} + \beta_{12}\{E[b(\tau)] - b(t)\} + \beta_{13}[C - r(t) \cdot s(t)] + \beta_{14}[A_1(t) - A_2(t)] \qquad (27)$$

where $A_1(t)$ = the aggregate amount of mortgages held plus forward commitments to borrowers.

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*Unlike the short hedger, for the long hedger, the partial derivative of $(h - i)/(u - i)$ with respect to $[C \cdot r(t) \cdot s(t)]$ is negative. This is so because h and l for the long hedger are negatives of h and l for the short hedger.

$A_2(t)$ = the aggregate amount of forward sales of mortgages plus [illegible] cash inflows to be invested.

Speculators

We now turn to speculator activity to determine the factors which affect the net supply of futures contracts. Speculators purchase or sell futures contracts to take advantage of differences between the expected futures price to prevail at a later date and the current futures price. As previously outlined, we will follow Stein's reasoning that most large speculators act in a risk-averse manner to attract capital for investment. We shall use the first three terms of a Taylor series expansion of a general utility function, omitting the fourth and subsequent terms as they are close to zero. This function approximates risk-averse behavior.

Let $A_0(t)$ = the speculating firm's initial wealth position at time t.

k = the number of futures contracts sold (positive or bought (negative).

So $[p(t) - p(\tau)] \cdot k$ is the speculator's profit from holding the short position of k contracts from time t_0 to τ.

We write the utility function as

$$U(A_0 + [p(t) - p(\tau)] \cdot k) = U(A_0) + U'(A_0)[p(t) - p(\tau)] \cdot k + \frac{U''(A_0)}{2}[p(t) - p(\tau)]^2 \cdot k^2 \quad (28)$$

The speculator will maximize the expected utility of wealth, which we can write as

$$E[U] = U(A_o) + U'(A_o)E[p(t) - p(\tau)] \cdot k + \frac{U''(A_o)}{2}E[p(t) - p(\tau)]^2 k^2 \qquad (29)$$

But, since

$$E[p(t) - p(\tau)]^2 = \sigma_p^2 + [p(t) - E[p(\tau)]]^2 \qquad (30)$$

we can write (24) as

$$E(U) = U(A_o) + U'(A_o)E[p(t) - p(\tau)] \cdot k + \frac{U''(A_o)}{2}k^2\{\sigma^2_p + [p(t) - E[p(\tau)]]^2\} \qquad (31)$$

The first order condition gives the optimal k^o, which is written as

$$k^o = \frac{-U'(A_o)\ [p(t) - Ep(\tau)]}{U''(A_o)\{\sigma^2_p + [p(t) - E[p(\tau)]]^2\}} \qquad (32)$$

The change in the optimal number of contracts sold when A_o increases is found from the expression

$$\frac{\delta k^o}{\delta A_o} = \frac{\delta}{\delta A_o}\left[\frac{-U'(A_o)}{U''(A_o)}\right] \cdot \frac{[p(t) - Ep(\tau)]}{\{\sigma^2_p + [p(t) - E[p(\tau)]]^2\}} \qquad (33)$$

We must assume that speculators will only sell contracts when they believe that $p(t) > E[p(\tau)]$. If speculators exhibit decreasing absolute risk aversion, then

$$\frac{d}{dA_o}\left[\frac{-U''(A_o)}{U'(A_o)}\right] < 0 , \tag{34}$$

and its inverse is greater than zero, making (33) greater than zero. Short speculators will increase their supply of futures when their initial asset bases increase.

It is possible to observe speculators buying contracts while others sell them. These long speculators are operating under the expectation that $p(\tau)$ will be greater than $p(t)$. In order to describe their behavior, the previous analysis can be repeated, replacing $\{p(t) - E[p(\tau)]\}$ with $\{E[p(\tau)] - p(t)\}$. We would again arrive at equation (33), with the change mentioned. Our conclusion is that long speculators will also increase demand for futures contracts as their initial asset bases increase. However, if there are more speculators short than long, we should expect net supply to increase when the aggregate initial wealth base increases.

The effect on the supply from an increase in the expected return per contract, $[p(t) - E[p(\tau)]]$, is found from the expression

$$\frac{\text{[illegible]}}{\partial\,[p(t) - E[p(\tau)]]} = -\left[\frac{U'(A_0)}{U''(A_0)}\right] \cdot$$

$$\left[\frac{[\sigma^2_p - [p(t) - E[p(\tau)]]}{\sigma^2 p + [p(t) - E[p(\tau)]]}\right] \quad (35)$$

The sign of (35) depends on the sign of $[\sigma^2_p - [p(t) - E[p(\tau)]]^2]$. If σ p is less than $[p(t) - E[p(\tau)]]$, then an increase in expected profit per contract will decrease the supply of futures by speculators. If σ_p is greater than $[p(t) - Ep(\tau)]]$, then an increase in expected profit will increase supply. We illustrate this result with the use of the graphs in Figures 2 and 3. We consider the simplest case of investing a fraction of each investment dollar in the riskless asset and the rest in futures. Suppose that we put up 100 percent margin on the sales. If the complete dollar is invested in the riskless asset, the net rate of return from time t to τ is i, represented by point i on the graphs. If the whole dollar is invested in the short futures position ($1/p(t)$ of a contract), the net expected rate of return is $[p(t) - E[p(\tau)]]/p(t)$, and the standard deviation is $\sigma_p/p(t)$. First, let this point be represented by X on the graph in Figure 2. The feasible set of portfolios formed by investing (or borrowing) at the riskless rate, i, and investing in futures for a total net investment of $1.00 is a straight line from i through X. Movement along the line from i represents an increased fraction of each dollar placed in the futures position. We can superimpose the speculator's indifference curves on the graph, shown here as curves I, II, and III. The speculator's optimal position is determined by the point of tangency of his indifference curves to the line iXA, say at X_1. We now consider the effect on the futures position held of an increase in the expected return from a futures contract, i.e., an increase in $\{p(t) - E[p(\tau)]\}$. We consider two new expected returns--one at Y and one at Z in Figure 3--and examine the effects of these increases in expected return on the fraction of each investment dollar placed in futures. Suppose, first, that Y is the point where $\{p(t) - E[p(\tau)]\} = \sigma_p$. We recall that this is an important point. For points above Y, $\sigma_p < \{p(t) - E[p(\tau)]\}$, and an increase in the expected return will decrease the number of futures contracts sold. For points below Y, the reverse is true.

We first examine the effect of an increase in expected return from X to Y. Since X is below Y, the increase causes an increase in the number of futures contracts sold. Turning to the graph, we see that the increase from X to Y shifts the feasible set from iXA to iYB. The new point of tangency to the difference curves is at X_2, and we can see that the

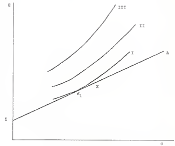

FIGURE 2
FEASIBLE PORTFOLIOS OF FUTURES
PLUS A RISKLESS ASSET

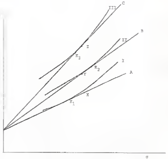

FIGURE 3
FEASIBLE PORTFOLIOS OF FUTURES
PLUS A RISKLESS ASSET

shift is in the direction of an increased short futures position for a given net investment; K_1 represented less than 100 percent of each dollar short in futures, while K_2 represents more than 100 percent (with the rest from borrowing) short in futures, since it is to the right of the point Y. Any further increase in the expected return from futures, say from Y and Z, will have the opposite effect. The feasible set now shifts to 1ZC in Figure 3, with the new point of tangency to indifference curve III at K_3. Since this point is to the right of Z, the speculator has returned to a position of less than 100 percent of his net investment in futures. Of course, the effect of a direct increase from X to Z is not as clear.

The fact that an increase in the expected rate of return from futures contracts may have two opposite effects, depending on the initial position can be attributed to the differing relative magnitudes of income and substitution effects at different points on the graph. We note first that an increase in the expected rate of return from futures is equivalent to a decrease in the expected futures price, since it then costs less to obtain the same expected future consumption from the short position. The substitution effect is, of course, always negative, and an increased expected rate of return will cause an increase in the short futures position on the compensated demand curve. However, in situations where $[p(t) - E[(T)]] > \sigma_p$ (and perhaps elsewhere), the income effect is positive. Here, it is positive enough to overcome the substitution effect and to result in a decrease in futures contracts sold when the expected return increases.

We note here that the decreased number of futures contracts sold with an increased expected return does not imply a lower total expected return to the speculator. In fact, it is evident from Figure 3 that the speculator may easily end up with a higher expected return after decreasing his futures position. So, while one might be led to say that futures contracts are an inferior good, we cannot draw the same conclusion about the expected future income which these contracts provide. Thus, the foregoing results are not as surprising as they might first seem.

Results from our sample would suggest that $\sigma_p > \{p(t) - E[p(\tau)]\}$ *, indicating that an increase in expected profit should increase speculators' net supply of futures. An aggregate function for the net supply of futures by speculators can now be written based on the preceding analysis.

We can now summarize the model which has been developed. The behavior of hedgers was first examined, resulting in a description of how an optimal hedge ratio would be determined. This description provided the determinants of the aggregate supply function were derived after considering how a

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*From Table 4-1, we see that the sample $\sigma_p = 2.958$, while on average, $\{p(t) - E[p(\tau)]\} = .099$ for the three year period.

typical speculator maximizes utility. The aggregate supply and demand functions are repeated below, followed by the limitations resulting from regulation of futures markets by the CFTC (described earlier).

$$D_t = \beta_{11} + \beta_{12}\{E[b(\tau)] - b(t)\} + \beta_{13}[C - r(t) \cdot s(t)] + \beta_{14}[A_1(t) - A_2(t)] \tag{35}$$

$$S_t = \beta_{21} + \beta_{22}\{p(t) - E[p(\tau)]\} + \beta_{23}A_0(t) \tag{36}$$

$$Q_t = \min(D_t\ S_t) \tag{37}$$

$$\left| p(t) - p(t-1) \right| = \min(\left| p^e(t) - p(t-1) \right|, L), \tag{38}$$

where $p^e(t)$ is the equilibrium price at which the quantity of futures demanded is equal to the quantity supplied.

We may make statements concerning the signs of the coefficients in the model of supply and demand--equations (35 and 36). The alternate hypotheses contain the predictions of the signs of the coefficients based on the model. If the model is not appropriate, then the null hypotheses should be appropriate. We list the sets of null and alternate hypotheses as follows:

Alternate Hypotheses	Null Hypotheses
$\beta_{12} > 0$	$\beta_{12} \leq 0$
$\beta_{13} < 0$	$\beta_{13} \geq 0$
$\beta_{14} < 0$	$\beta_{14} \geq 0$
$\beta_{22} > 0$	$\beta_{22} \leq 0$
$\beta_{23} > 0$	$\beta_{23} \leq 0$

CHAPTER IV
THE SAMPLE AND PRELIMINARY RESULTS

A description of the sample of interest rate futures and the definitions of the variables used in this study are presented in this chapter. The results of preliminary tests of the hypotheses are also discussed.

The Sample

The interest rate futures contract studied is the contract specifying delivery of the Collateralized Depository Receipt for GNMA's described in Chapter I, which has been traded at the Chicago Board of Trade since October, 1975. The 740 daily observations on price and quantity used in the current study cover the time period from January 1, 1976, through December 31, 1978, the first three calendar years in which this type of contract existed.

More than one GNMA futures contract exists at any one time, since contracts specifying that delivery be made in different months do exist simultaneously. At any given time, there are usually GNMA futures contracts with eight or more delivery dates actively traded at the Chicago Board of Trade, of which any one delivery date might be chosen as representative. If contracts with one delivery date were to provide investors with a better price or correlation with the corresponding GNMA certificate than the others, then arbitrage should occur. In this study, the GNMA futures contract chosen for a given day is the one having the closest delivery date without being in the month of the date of delivery. For example, the September 1977 contract (the contract which specifies delivery in September 1977) is chosen for the daily observations for June, July, and August of 1977, while the December 1977 contract is chosen for the daily observations in September, October, and November of 1977. This contract consistently has one of the two highest volumes of sales and open interest for the period studied.* Thus it should reflect the consensus of many buyers and sellers of GNMA futures contracts.

Days of disequilibrium are defined as those days in which the price of the contract reaches the upper or lower limit allowed by the Commodity Futures Trading Commission for the day and closes at that limit. For example, if the price reaches the upper limit of the day and closes at that upper limit, that day is a day of disequilibrium resulting from excess demand. If, on the other hand, the price reaches the upper limit and falls to a lower settlement price, the day is not a day of disequilibrium. Of the sample of 740 days, the price reached a lower limit on five of the days, and an upper limit on three of the days.

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*Information on volume of contracts traded was taken from the Chicago Board of Trade Statistical Annual for 1976, 1977, and 1978.

Definition of Variables

Q_t. The number of futures contracts traded during day t for the contract being studied is represented by Q_t. If the price closes at the upper limit on day t, the transactions are assumed to represent the supply at that price, since there will be excess demand. If the price closes at the lower limit, the transactions are assumed to represent the demand at that price, since there will be excess supply. Q_t is assumed to be the equilibrium quantity on all other days. The data for Q_t are published in the yearly Chicago Board of Trade Statistical Annual.

$p(t)$. The last price for day t for the contract being studied is represented by $p(t)$. These prices are obtained from the Chicago Board of Trade Statistical Annual.

$s(t)$. Because the futures contract studied is that which calls for delivery of the equivalent of GNMA certificates with 8 percent interest, the spot price for day t, $s(t)$, is the price of GNMA 8 percent certificates. Neither the bid price nor the asked price would apply exclusively to either hedgers or speculators, since both groups of traders contain both buyers and sellers. The spot price, $s(t)$, is therefore computed as the average of the bid and asked prices, as quoted in the Wall Street Journal for day t.

Time τ is defined in Chapter III as the time horizon of the firm or the individual hedger or speculator. Any of these traders may have relatively long periods over which to invest, but all would continue to monitor daily price movements in order to reevaluate decisions to hold positions. Therefore, even if traders plan to hold positions in a contract over several months, the proper time horizon over which they make decisions to buy and sell is one day. The time horizon, τ, is therefore defined as (t+1).

$E[s(\tau)]$. This variable represents the spot price which is expected to prevail at day τ, where $\tau = t + 1$. Two basic techniques were considered before choosing a method to measure market expectations of subsequent prices. The first technique involves the use of distributed lags, as suggested by Almon (1965) and others. In the context of this study, a distributed lag of spot prices might be used in a regression for the expected spot price at day (t + 1). This could be written as

$$E[s(t + 1)] = \beta_0 s(t) + \beta_1 s(t - 1) + \ldots + \beta_n s(t - n) \quad (1)$$

where the β's follow a polynomial, such as

$$\beta_i = \alpha_0 + \alpha_1 i + \alpha_2 i^2$$

The use of a distributed lag model to represent expected prices would be plausible if investors form expectations based not only on the current price, but also on prices which occurred in the recent past. Implications of using this model are that prices follow a discernible pattern over time, that investors know the pattern well enough to make predictions, and that pattern is given by a well-defined function such as the one above. While the distributed lag is sometimes used as a measure of expectations of different variables, there are several limitations associated with its use. As Maddala (1977) has indicated, there is no way of knowing how long the lag should extend, and it is not even possible to test for the presence of any lag. Furthermore, there is no economic theory to indicate that the spot price of GNMA certificates should follow any such pattern. Distributed lag models were therefore rejected as being inappropriate for this study.

The technique which was chosen to measure the expected spot price is a more simple prediction model of the autoregressive type. If the current spot price reflects all publicly available information relevant to the pricing of GNMA certificates, then the current spot price should be the best predictor of the next day's spot price. Based on this hypothesis, a prediction model was formulated as

$$E[s(t + 1)] = \alpha_0 + \alpha_1 s(t). \qquad (2)$$

The values of α_0 and α_1 were estimated by regressing s(t) on s(t - 1), and substituting the ordinary least squares [illegible] in equation [illegible]. The predictions were [illegible] measured as

$$[illegible] \qquad (3)$$

The estimates for β_0 and β_1 are given in equation (4). Standard errors of the estimates are presented in parentheses.

$$s(t) = -0.0519 + 1.0004s(t-1), \quad R^2 = 0.9935$$
$$(0.2884) \quad (0.0029) \qquad DW = 2.1585 \qquad (4)$$

Maddala (1977) discusses Theil's (1966) statistics which give a measure of the accuracy of a prediction model such as the one in equation (3). They are

$$U^M = \frac{(\bar{P}-\bar{A})^2}{MSE}, \qquad (5)$$

$$U^R = \frac{(S_p - rS_a)^2}{MSE}, \qquad (6)$$

and

$$U^D = \frac{(1-r^2)S_a^2}{MSE}. \qquad (7)$$

A_t represents the actual spot price on day t and P_t represents spot price for day t as predicted by equation (3), and

$$a_t = (A_t - A_{t-1})/A_{t-1} , \qquad (8)$$

$$p_t = (p_t - A_{t-1})/A_{t-1} , \qquad (9)$$

S_p, S_a = the standard deviations of p and a , (10)

$$\text{MSE} = \text{the mean square error} = 1/n \sum_{t=1}^{n} (p_t - a_t)^2 , \qquad (11)$$

r = the correlation between p and a, (12)

and $\bar{p}$ and $\bar{a}$ are the sample means of p and a, respectively. Of the above statistics, u^M and u^R give the most information about the accuracy of the prediction model. The bias proportion of the mean square error, u^M, is a measure of the magnitude of the difference between the average predicted and the average actual changes in the variable. The bias proportion would equal zero if the mean of the actual change were equal to the mean of the predicted change. The regression proportion of the mean square error, u^R, gives a measure of the difference in the dispersions of the actual and the predicted changes. The statistic u^R would increase with an increase in the difference between the standard deviation of a_t and p_t, or a decrease in the correlation between them. If a_t and p_t had the same standard deviations and were perfectly positively correlated, u_R would equal zero. If both u^M and u^R are close to zero, the predicted changes are close to the actual changes in terms of mean, standard deviation, and correlation, and the prediction model is quite accurate. In such a case there is no systematic over- or under-prediction of the change, and the variance of the predicted and actual changes are close in magnitude. One way to measure the statistics in (5), (6), and (7) is to examine the results of the regression of the actual variables on the corresponding predictions. In this case, we let $E[s(t + 1)]$ be represented by P_t and the actual spot price on day t, s(t), be represented by A_t. The variable A_t is then regressed on P_t, written as

$$A_t = \delta_0 + \delta_1 P_t + V_t . \tag{13}$$

The ordinary least squares estimates, $\hat{\delta}_0$ and $\hat{\delta}_1$, provide information about u^M and u^R. First, if $\hat{\delta}_1 = 1$, then $u^R = 0$. Equation (13) can be rewritten by dividing through by A_{t-1} as

$$\frac{A_t}{A_{t-1}} = \delta_0 \frac{1}{A_{t-1}} + \delta_1 \frac{P_t}{A_{t-1}} \quad , \quad A_{t-1} \neq 0 \tag{14}$$

From (14), the OLS estimate of δ_1 is

$$\hat{\delta}_1 = \frac{\sum_{t=1}^{n} \left[\frac{P_t}{A_{t-1}} - 1/n \sum_{t=1}^{n} \frac{P_t}{A_{t-1}} \right] \cdot \left[\frac{A_t}{A_{t-1}} - 1/n \sum_{t=1}^{n} \frac{A_t}{A_{t-1}} \right]}{\sum_{t=1}^{n} \left[\frac{P_t}{A_{t-1}} - 1/n \sum_{t=1}^{n} \frac{P_t}{A_{t-1}} \right]^2} \tag{15}$$

If $\hat{\beta}_1 = 1$, then (15) indicates that

$$\sum_{t=1}^{n}\left[\frac{P_t}{A_{t-1}} - 1/n \sum_{t=1}^{n} \frac{P_t}{A_{t-1}}\right] \cdot \left[\frac{A_t}{A_{t-1}} - 1/n \sum_{t=1}^{n} \frac{A_t}{A_{t-1}}\right]$$

$$= \sum_{t=1}^{n}\left[\frac{P_t}{A_{t-1}} - 1/n \sum_{t=1}^{n} \frac{P_t}{A_{t-1}}\right]^2 \qquad (16)$$

But

$$S_p^2 = \sum_{t=1}^{n}\left[\frac{P_t-A_{t-1}}{A_{t-1}} - 1/n \sum_{t=1}^{n} \frac{P_t-A_{t-1}}{A_{t-1}}\right]^2$$

$$= \sum_{t=1}^{n}\left[\frac{P_t}{A_{t-1}} - 1/n \sum_{t=1}^{n} \frac{P_t}{A_{t-1}}\right]^2 \qquad (17)$$

and

$$S_{ap} = \sum_{t=1}^{n}\left[\frac{P_t-A_{t-1}}{A_{t-1}} - 1/n \sum_{t=1}^{n} \frac{P_t-A_{t-1}}{A_{t-1}}\right] \cdot \left[\frac{A_t-A_{t-1}}{A_{t-1}} - 1/n \sum_{t=1}^{n} \frac{A_t-A_{t-1}}{A_{t-1}}\right]$$

$$= \sum_{t=1}^{n}\left[\frac{P_t}{A_{t-1}} - 1/n \sum_{t=1}^{n} \frac{P_t}{A_{t-1}}\right] \cdot \left[\frac{A_t}{A_{t-1}} - 1/n \sum_{t=1}^{n} \frac{A_t}{A_{t-1}}\right] \qquad (18)$$

We see that (17) is the same as the right hand side of (16), and (18) is equal to the left hand side of (16). Therefore, if $\hat{\beta}_1 = 1$, then equation (17) equals equation (18), or

$$S^2_p = S_{ap}, \qquad (19)$$

or

$$S_p = rS_a \qquad (20)$$

Therefore, from (20) and (6), we see that if $\hat{\beta}_1 = 1$, then

$(S_p - rS_a$ [illegible] the regression proportion, U^R, is equal to zero.

Returning, to the OLS estimate of δ_o, we shall see that if $\hat{\delta}_o = 0$ and $\hat{\delta}_1 = 1$, then $U^M = 0$. From (14), the OLS estimate of δ_o is

$$\hat{\delta}_o = 1/n \sum_{t=1}^{n} \frac{A_t}{A_{t-1}} - \hat{\delta}_1 \; 1/n \sum_{t=1}^{n} \frac{P_t}{A_{t-1}} \tag{21}$$

If $\hat{\delta}_o = 0$ and $\hat{\delta}_1 = 1$, then

$$1/n \sum_{t=1}^{n} \frac{A_t - P_t}{A_{t-1}} = 0 \tag{22}$$

But

$$\begin{aligned} \bar{p} - \bar{a} &= 1/n \sum_{t=1}^{n} \left[\frac{P_t - A_{t-1}}{A_{t-1}}\right] - 1/n \sum_{t=1}^{n} \left[\frac{A_t - A_{t-1}}{A_{t-1}}\right] \\ &= 1/n \sum_{t=1}^{n} \left[\frac{P_t - A_t}{A_{t-1}}\right]. \end{aligned} \tag{23}$$

From (22) and (23), we see that when $\hat{\delta}_o = 0$ and $\hat{\delta}_1 = 1$, that $(\bar{p} - \bar{a})^2 = 0$ and the bias proportion, given by equation (5), is equal to zero.

The regression in (13) was run in order to have some idea of the U^M and U^R for the prediction model for $E[s(t + 1)]$. The OLS estimates are given below, with the

[illegible]

$$\hat{s}_t = 2.879 \times \text{[illegible]} + \text{[illegible]} \qquad (24)$$
$$(0.282) \qquad \text{[illegible]}$$

It is clear from (24) that $\hat{\alpha}_0$ is not significantly different from zero, and $\hat{\beta}_1$ is not significantly different from one at the 99% level of confidence, indicating that both [illegible] and u^M are approximately equal to zero for the model. It was therefore concluded that the prediction model for E[s(t + 1)] using the $\hat{\alpha}_0$ and $\hat{\beta}_1$ given by (4) is relatively accurate, based on the criteria of small bias and regression proportions. The variable of E[s(t + 1)] is thus measured as

$$E[s(t + 1)] = -0.0519 + 1.0004s(t) \qquad (25)$$

E[p(τ)]. The futures contract price which traders expect to occur on day τ is written as E[p(τ)], where τ = (t + 1). A prediction model of an autoregressive form was used to measure E[p(t + 1)].

There are conditions under which E[p(t + 1)] is greater than p(t) by a given amount. The relationship between p(t) and s(t) will first be developed and then used to find the expected price at time (t + 1) of the futures contract. We consider an investor who can borrow s(t) dollars at a daily rate of i for the time period of (T-t). With the money, the investor buys a GNMA certificate at price s(t) which pays c · (T-t) dollars in interest over time period, and simultaneously sells a futures contract for p(t) which specifies delivery at time T of the GNMA certificate being held. At time T, the investor can deliver the certificate on the futures contract for the price of $p(t)$, repaying the loan with the amount $s(t)[1 + i \cdot (T-t)]$. Over the time period of t to T, the investor has a zero net investment, since the money invested in the certificate was borrowed. The investor must put up a small amount of margin for the futures contract with the broker, but the opportunity cost of the margin is negligible because interest-bearing securities can be used, allowing the investor to earn interest on his margin. The cost to the investor at time T is $s(t)[1 + i \cdot (T-t)]$ and the receipts are $p(t)$ and $c \cdot (T-t)$. The investor will therefore compare the cost with the receipts at time T to see if there is an opportunity to earn a profit on a zero net investment. If the receipts of $[p(t) + c(T-t)]$ are greater than the cost of $[s(t) + i \cdot (T-t) \cdot s(t)]$ at time T, that is if

$$p(t) + c \cdot (T-t) > s(t) + i \cdot (T-t) \cdot s(t) \qquad (26)$$

then the investor will buy the GNMA certificate and sell the futures contract to earn the profit from a zero net investment. Investors will continue to arbitrage in this way until the inequality in (26) no longer holds.

There is an alternative strategy which investors can follow to earn a profit from a zero net investment when the inequality in (26) is reversed to

$$p(t) + c \cdot (T-t) < s(t) + i \cdot (T-t) \cdot s(t). \qquad (27)$$

At time t, the investor makes a loan of s(t) dollars for the time (T-t) at a daily rate of i to an owner of a certificate, taking the certificate as collateral. The investor then sells the certificate for s(t) and simultaneously buys a futures contract specifying the delivery of a certificate at time T for p(t) dollars. At time T, the investor takes delivery of the certificate, paying the price p(t). This certificate is then returned to the original owner, who pays off the loan plus interest to the investor. Finally, the investor must pay the original owner the coupon payments of c · (T-t) which have accrued over the time that the certificate was being held as collateral. The investor has had no investment from time t to T, since the loan of s(t) dollars was covered by the s(t) dollars from the sale of the certificate. At time T, the investor receives s(t) + i · (T-t) · s(t) from the repayment of the loan, and pays p(t) for the certificate to be returned to the borrower, plus c · (T-t) in coupon payments, which are also paid to the borrower. If (27) holds, then the investor's receipts are greater than the payments, and he has earned a profit from a zero net investment. As long as (27) holds investors will continue to follow this strategy, driving down the price s(t) as certificates are sold and driving up p(t) as futures contracts are bought. This arbitrage will continue until the inequality in (27) no longer holds.

Since arbitrage will continue until neither (26) nor (27) hold, the equilibrium relationship under these circumstances is

$$p(t) + c(T-t) = s(t) + i \cdot (T-t) \cdot s(t), \qquad (28)$$

or

$$p(t) + [c \cdot (T-t) - i \cdot (T-t) \cdot s(t)] = s(t). \qquad (29)$$

The above expression can be rewritten for the time (t+1), where (t+1) < T as

$$p(t + 1) + [c \cdot (T-(t + 1) - i \cdot (T - (t + 1) \cdot s(t + 1)] = s(t + 1) \qquad (30)$$

Taking the expected value of (30), we get

$$E[(p(t + 1)] + [c \cdot (T-t - 1] - i \cdot (T-t - 1) \cdot E[s(t + 1)]] = E[s(t + 1)] \qquad (31)$$

Recalling the results given in equation (4), we use the approximation

$$E[s(t + 1)] \approx s(t).$$

Substituting this into [illegible], we get

$$E[p(t$$ [illegible] $$T - t - 1] - i \quad [T - t -$$ [illegible]

[illegible] (32)

Subtracting (29) from [illegible] yields

$$E[p(t + 1)] - p(t) + [c \quad (-1) - i \cdot (-1)$$ [illegible]
$$= 0, \qquad (33)$$

or

$$E[p(t + 1)] = p(t) + [c - i \cdot s(t)] \qquad (34)$$

The expected futures price at time (t + 1) should equal the previous day's futures price plus the day's coupon payment, c, less the one day's cost of borrowing the money to buy the security, i. Equation (34) was used to formulate a prediction model for $E[p(t + 1)]$.

as

$$E[p(t + 1)] = \beta_0 + \beta_1 p(t) + \beta_2 [c - i \cdot s(t)] \qquad (35)$$

The next steps in formulating and evaluating the prediction model for $E[p(t + 1)]$ are identical to those for the prediction model for $E[s(t + 1)]$. The values of β_0, β_1 and β_2 were estimated as the ordinary least squares estimates, $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\beta}_2$ from the following regression.

$$p(t) = \beta_0 + \beta_1 p(t - 1) + \beta_2 [c - [illegible] s(t)] + u_t$$

The estimates are given in equation (37), with their standard errors given in parentheses.

$$p(t) = 0.9587 + 0.9890p(t - 1) - 13.2111[c - [illegible]]$$

$$(0.4876)\ (0.0054) \qquad (4.4108)$$

$$R_2 = .9923 \qquad (37)$$

$$DW = 2.0053$$

These ordinary least squares estimates were then substituted in for β_0, β_1, and β_2 in (35) to measure $E[p(t + 1)]$.

The accuracy of this prediction model was then tested using the techniques described for the $E[s(t + 1)]$ prediction model. The actual price, $p(t + 1)$, was regressed on $E[p(t + 1)]$ as given by equation (35). The regression can be written as

$$A_t = \lambda_0 + \lambda_1 p_t + w_t \qquad (38)$$

We recall from the discussion of $E[s(t + 1)]$ that the OLS, $\hat{\lambda}_0$ and $\hat{\lambda}_1$, provide information about the bias proportion and the regression proportion of the mean square error. Specifically, if $\hat{\lambda}_0 = 0$ and $\hat{\lambda}_1 = 1$, then both the bias proportion and the regression proportion are zero and the prediction model is quite accurate.

The regression in (38) resulted in the OLS estimates $\hat{\lambda}_0$ and $\hat{\lambda}_1$, given in (39), with the standard errors of [illegible] estimates given in parenthesis.

$$A_t = 1.7376 \times 10^{-11} + 1.000\ p_t \qquad (39)$$
$$(0.3098) \qquad\qquad (0.0032)$$

From the results given in (39), we see that $\hat{\lambda}_0$ is not significantly different from zero and $\hat{\lambda}_1$ is not significantly different from 1.0 at the 99% level of confidence, indicating that the bias and regression proportions are zero. Based on this criteria, the prediction model is assumed to be quite accurate. The variable $E[p(t + 1)]$ is therefore measured as

$$E[p(t + 1)] = 0.9587 + 0.989p(t)$$
$$+ 13.2111[c - i \cdot s(t)]$$

$\underline{A_0(t)}$. The variable $A_0(t)$ represents the average size of the asset base of speculators in GNMA contracts at time t. A survey by Bobeon (1978) shows that the largest proportion of speculators who are classified as commercial traders are securities dealers. The average asset size of securities dealer firms therefore may provide a proxy for the asset size of most of the speculating participants.

The trading firm which speculates in GNMA futures is assumed to contain certain dealers who perform most of the speculative trading. At the same time, other of the firm's dealers are assumed to trade in government securities such as mortgages and Treasury bills and Treasury bonds. Gains or losses from trading of these government securities will therefore increase or decrease the size of the asset base itself. As proceeds from trades are reinvested in securities, the dollar value of the inventory of government securities held by dealer firms should reflect the size of their asset base.

Therefore, the average size of the asset base at time t for speculators was estimated as the daily average dollar value of United States government securities held by securities dealers. The par value of the daily average dealer position for each month of the time period studied was taken from the monthly Federal Reserve Bulletin. The data are listed by maturity classification, with maturities of less than one year, one to five years, five to ten years, and over ten years for the government securities. United States government agency security positions are also listed. These data were used in conjunction with yields and price quotations for United States government and government agency securities to estimate the daily average market value of dealer's inventories of all such securities. In order to do this estimation, a market value per unit of par value was first derived for each category.

The average daily market value of securities maturing in less than one year was estimated by assuming an average maturity of six months and then applying the average six month yield for each week to the corresponding average daily par value of those securities. The average yield for six month securities was obtained from the Federal Reserve Bulletin, which lists the average of the closing bid yields for six-month Treasury Bills, for each week of the period. The average daily market value for such securities for each week was then found by applying the formula of

$$M.V. = P.V. \; (1-y) \qquad (43)$$

where M.V. = average daily market value

P.V. = average daily par value

y = average bid yield, as quoted in the Federal Reserve Bulletin.

The average daily market values of government and government agency securities with longer maturities were derived in a different manner. For these securities, price quotations were collected from the Wall Street Journal for Wednesday's trading of each week (unless the markets were closed on that day, in which case the quotations for Thursday were used). These price quotations were divided into groups by the categories previously described. Within each group, all bid quotations listed in the Wall Street Journal were added and averaged. The resulting figures represented, for each category, the average bid price on that day for all outstanding securities in the category. Each average bid price was then used as the daily average over the week for the category. The average daily market value for the week for each category was then calculated by using the formula of

$$\text{M.V.L.} = \text{P.V.L.} \times \text{B.Q.} \qquad (41)$$

where M.V.L. = average daily market value of securities within a specified category

P.V.L. = average daily par value of the securities within the specified category which were held by dealers

and

B.Q. = average bid price for all securities listed in the Wall Street Journal within the specified category.

For a given week, the resulting market values for each category were added together, and the total was used as the average daily size of the asset base of the speculators for that week.

$\underline{A_1(t)}$. This variable represents new mortgages and the increase in outstanding commitments to borrowers over the previous period.

Mortgage banks and savings and loan institutions may sell GNMA futures contracts to hedge against rises in interest rates while they are acquiring a pool of mortgages which are all guaranteed by the Federal Housing Authority or the Veterans Administration. Although interest rates for the mortgages in the pool have already been negotiated, GNMA certificates representing shares of the pool cannot be sold until approval from the Government National Mortgage Association has been granted. A rise in interest rates during the acquisition of the pool could cause a decline in the market value of the mortgages before the GNMA certificates can be sold, thereby resulting in a loss to the institution. By selling GNMA futures contracts, the institution can attempt to offset such a loss. The size of the pool of mortgages which has not already been sold as GNMA certificates is an indication of the number of futures contracts which could be sold as a hedge against such a loss for any period.

The total size of the mortgage pools being accumulated but not yet sold for each month was estimated as follows. The monthly dollar amount of mortgages by mortgage banks for buildings of one to four units plus loans closed by savings and loan institutions was found in The Mortgage Banker. To this monthly figure was added the increase from the previous month in outstanding commitments to borrowers from mortgage banks and savings and loan institutions. The resulting figure gives the monthly net increase in commitments and new mortgages for the institutions described. The increase in commitments was used rather than the total commitments for the month in order to avoid double counting. For example, a firm may make a commitment this month and two commitments next month. If the original commitment is transformed into a loan during the next month, then counting the loan plus all three commitments would be a partial double counting over the two months. The issuing of the loan represents an offsetting decrease in commitments. Therefore, the amount of loans plus the increase in commitments for a month provides the net increase of such financing.

The dollar value of mortgages which were sold during a month as GNMA certificates was taken from The Mortgage Banker, which lists the value of all GNMA certificates issued during each month. The resulting figure for the month was subtracted from the corresponding net increase in loans and commitments, yielding a monthly indication of the number of futures contracts which could be sold to hedge against the new unsold mortgage pools.

In summary, $A_1(t)$ is a monthly figure representing the dollar value of loans closed plus increases in the value of outstanding commitments to borrowers for mortgage banks and savings and loan institutions, minus the value of GNMA certificates sold.

$A_2(t)$. This variable represents the increase in forward sales of mortgages over the month plus expected cash to be invested for the next quarter by financial institutions.

The increase in forward sales of assets is measured as the monthly increase in commitments to mortgage banks and savings and loan institutions from investors. This monthly figure is found in The Mortgage Banker.

Expected levels of cash were measured on a quarterly basis since no data were available to yield a figure for more frequent intervals. The data used were based on forecasts prepared by Chase Econometrics. For each month, the forecasts of demand deposits, government demand deposits, consumer time deposits, certificates of deposit, and thrift deposits for the following quarter were added together. The resulting figure represents the forecast, based on information available during that month, of cash which could be invested over the next quarter. Under the assumption that financial institutions make estimates of cash levels which are based on the same information as that available to Chase Econometrics, this forecast data should be indicative of firms' expectations about subsequent levels of cash to be invested. Thus, the monthly figure for the $A_2(t)$ variable is composed of the monthly Chase Econometric forecast of the next quarter's deposits plus the monthly increase in commitments from investors to mortgage banks and savings and loan institutions.

$\underline{b(t)}$. The basis, $b(t)$, is defined as the futures price minus the spot price on day t. So,

$$b(t) = p(t) - s(t) \ . \tag{42}$$

$\underline{E[b(\tau)]}$. The expected basis at time τ, where $\tau = (t+1)$ is defined as the difference between the expected futures price minus the expected spot price at time $(t+1)$. So

$$E[b(\tau)] = E[p(\cdot)] - E[s(\tau)] \tag{43}$$

where $E e[p(\ )]$ and $E[s(\ )]$ are measured in the manner already described.

$C_{t,T}$ is the coupon payment per $100 par value on a GNMA certificate for the time remaining from day t for a hedger. It is assumed that the short hedger holding the certificate who uses the futures contract maturing on day T continues to hold the certificate, on average, from time t to the time of maturity of the futures contract, day T. While some hedgers may hold the certificate for less than this time, others may hold it for longer. However, if a hedger were planning to hold the GNMA certificate for a length of time much past day T, he or she would use the futures contract with a longer maturity to avoid the need to sell a second contract once day T has passed, since the second contract would mean additional transactions costs and time. Therefore, it is reasonable to assume that short hedgers using the futures contract maturing on day T continue to earn the coupon payment from time t to T, on average. The time period (T-t) is also the relevant one for long hedgers who are considering the foregone coupon payments as an opportunity cost. The long hedger will only buy the contract maturing on day T if that maturity date most closely matches the date of the hedger's expected cash investment. If the investment is expected to occur long enough after day T, a long hedger holding that contract would have to buy a second contract maturing later--incurring additional transactions costs--in order to maintain the hedge. For these reasons, the time period of the coupon payment is calculated as the number of days remaining on the futures contract, (T-t), and C is calculated as the daily payment per

$100 multiplied by the time period, (T-t). Since the yearly payment is eight percent, then

$$C = 8/365 \cdot (T-t) \tag{44}$$

r(t). The rate at which a hedger could borrow money to buy a GNMA certificate is r(t). For the reasons discussed in the preceding paragraphs, the appropriate time period is (T-t) days. Data on interest rates for each a specific time period are not available. However, data for the overnight repurchase rate--the rate at which an investor may borrow funds overnight using a government security as collateral--were used to obtain an approximation of r(t). The overnight repurchase rate for day t, i*, was multiplied by the number of days in the time period (T-t), and the result was used as an estimate of r(t) for day t.

The measurement of each variable contained in the supply and demand equations from Chapter III has now been described. Sample statistics for the preceding variables are presented in Tables 4-1, 4-2 and 4-3. The hypotheses concerning the sign of each variable are tested by applying econometric estimation techniques to each equation. In the absence of price regulation, two stage least squares (2SLS) would be the appropriate technique to estimate the supply and demand equations since the price and quantity of futures contracts would be simultaneously determined in the two equations. However, whenever the price limit is met, the futures price variable switches from being simultaneously determined with quantity (endogenous), to being exogenous. Unlike other models in which the switch is determined by forces outside the market, the nature of the futures price variable is determined by forces within the market. It is the extent of the change in the equilibrium price which determines whether price is exogenous or endogenous. Therefore, the standard switching equation techniques cannot be applied to this market.

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*The overnight repurchase rates were taken from Data Resource Inc., "Yellow Pages of Rates."

The remainder of this chapter contains the preliminary results obtained from the OLS and 2SLS estimation techniques. The correct estimation technique is discussed in Chapter 5.

Estimation by Ordinary Least Squares

The demand and supply equations were first estimated using the OLS technique. If the futures price had reached an upper or lower limit for each observation in the sample, then price would not have been determined simultaneously with quantity for any observation, and OLS would have been the correct estimation procedure. Since the price limit was only reached during eight of the 740 days in the study, price is determined simultaneously with quantity for 732 of the days. Therefore, the price variable is correlated with the error term in both the supply and demand equations, making OLS estimates inconsistent. However, OLS estimation was done for both the supply and demand equations for the purpose of

TABLE 4-1
DESCRIPTIVE STATISTICS FOR THE SAMPLE VARIABLES

Variable	Mean	Standard Deviation	Maximum	Minimum
Q	527.7986	502.8347	4000.0	14.000
$p(t)$	95.5615	2.9581	101.125	87.719
$E[p(\tau)]$	95.5505	2.9602	101.091	87.682
$s(t)$	96.9517	3.0217	103.25	89.875
$b(t)$	-1.3902	0.5671	-0.093	-2.969
$E[b(\tau)]$	-1.3911	0.5590	-0.108	-2.937
$[C-r(t)a(t)]$	0.4281	0.2899	1.188	-0.245
$[A_1(t)-A_2(t)]$	5863.0026	1560.8330	93132.200	2514.500
$A_0(t)$	5682.2802	276.1525	12611.200	-186.900

TABLE 4-2

CORRELATION COEFFICIENT MATRIX FOR THE SAMPLE VARIABLES

	Q	p(t)	s(t)	b(t)	[C-r(t)s(t)]	[A_1(t)-A_2(t)]	A_o(t)
Q	1.0						
p(t)	-0.513	1.0					
s(t)	-0.514	0.982	1.0				
b(t)	0.067	-0.017	-0.205	1.0			
[C-r(t)s(t)]	-0.398	-0.589	0.665	-0.474	1.0		
[A_1(t)-A_2(t)]	0.165	-0.051	-0.056	0.031	-0.145	1.0	
A_o(t)	-0.442	0.678	0.731	-0.359	0.608	-0.343	1.0

TABLE 4-3
COVARIANCE MATRIX FOR THE SAMPLE VARIABLES

	Q	p(t)	s(t)	b(t)	(C-r(t)s(t))	$[A_1(t)-A_2(t)]$	$A_0(t)$
Q	252,843.00						
p(t)	-762.65	8.7505					
s(t)	-781.677	8,780.0000	9.131				
b(t)	19.027	-0.0290	-0.351	0.322			
[C-r(t)s(t)]	-57.956	0.5050	0.583	-0.078	0.084		
$(A_1(t)-A_2(t))$	129,557.000	-253.2380	-262.810	27.572	-65.690	243,620	
$A_0(t)$	-614,485.000	5,547.5600	6,111.610	-564.049	488.060	-1,490,865	7,657,144

comparison with subsequent estimates. The results are summarized in Table 4-4.

From these estimates, it would appear that both the demand and supply equations are upward-sloping. The coefficient of the price variable is significantly greater than zero in both equations,* indicating that hedgers would increase their demand for futures contracts with an increase in the current futures price. Since the hedger's expected profit from a long hedge when assets are hedged completely is $\{E[b(\tau)] - b(t)\}$, the sign of the coefficient indicates that the hedger decreases demand for futures contracts when there is an increase in expected profit. This result is not consistent with the hedging behavior predicted from the model in Chapter III.

The coefficient of the $[C - r(t)s(t)]$ variable is significantly less than zero, as predicted from the model, but the $[(A_1(t) - A_2(t)]$ and the $A_0(t)$ coefficients are not significantly different from zero.

Thus, we could only reject the null hypothesis for the d $[C - r(t)s(t)]$ and $\{p(t) - E[p(\tau)]\}$ variables if we were to rely on the OLS results.

Estimation by Two Stage Least Squares

Under ordinary circumstances, the estimation of a supply or demand equation and the price variable of the supply equation is done using the 2SLS technique. This is the appropriate method when the observations are from a market in

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*Since the coefficient of $\{E(b(\tau)] - b(t)\}$ is negative and $b(t) = p(t) - s(t)$, the coefficient of the price variable, $p(t)$, is positive.

TABLE 4-4
RESULTS FROM ESTIMATION BY ORDINARY LEAST SQUARES

A. Demand Equation

$R^2 = 0.2164$

Variable	Intercept	$\{E[b(\tau)]-b(t)\}$	$[C-r(t)a(t)]$	$[A_1(t)-A_2(t)]$
estimated coefficient:	641.920	-4909.797	-454.711	0.013
t:	(8.880)	(-6.614)	(-6.972)	(1.164)

B. Supply Equation

$R^2 = 0.3023$

Variable	Intercept	$[p(t)-E[p(\tau)]]$	A_0t
estimated coefficient:	826.182	6434.84	0.065
t:	(21.525)	(10.647)	(-11.236)

which the price is allowed to adjust to that level where the quantity demanded is equal to the quantity supplied. In such a market, price and quantity are determined simultaneously, and the price variable is correlated with the error term in both the supply and demand equations. The correlation of the price variable and the error term in the equations violates the assumption of a zero covariance between regressors and the error term, an assumption which must be met before we can expect OLS estimates to be consistent. Since the estimates of the coefficients obtained from OLS regression for the supply and demand equations are inconsistent due to this correlation, the use of a different technique which provides consistent estimates becomes necessary. The method of 2SLS provides consistent estimates by first replacing the price variable with a related variable having no correlation with the error term in each equation. As its name suggests, 2SLS estimation proceeds in two steps. In the first step, price is regressed against all of the exogenous variables--variables which are not simultaneously determined with price and quantity--using OLS to obtain an expression for price, $\hat{p}$, in terms of the exogenous variables. This new variable, $\hat{p}$, is related to the price variable but it is independent of the error terms in the supply and demand equations. The second step of the 2SLS procedure is to replace p with $\hat{p}$ in the supply and demand equations, using OLS estimation on the new set of variables. The resulting estimates of the coefficients are consistent for markets in which price and quantity are always simultaneously determined. The estimated coefficients are presented in Table 4-5.

Examining the estimates for the demand equation, we see from the t-statistics, given in parentheses, that we cannot accept the null hypotheses as stated concerning the basis, $b(t)$, the expected basis, $E[b(\tau)]$, and the earnings foregone by hedgers, $[C-r(t)\cdot s(t)]$. Unlike the OLS results, the 2SLS estimate of the coefficient of $\{E[b(\tau)] - b(t)\}$ is significantly greater than zero, indicating that if hedgers are net long, they increase their demand for futures contracts as the basis, $p(t) - s(t)$, decreases. The demand curve for futures contracts is downward sloping with respect to the futures price. An increase in the expected basis results in an increase in the expected return to long hedgers, and the 2SLS estimates indicate that hedgers respond to this increase by increasing demand for futures contracts. The amount $[C - r(t)s(t)]$ may be looked upon as a type of fixed cost to the potential long hedger of not holding the security until a later date. We recall from the model that a decrease in this fixed cost increases the optimal fraction of the expected investment to hedge. The results in Table 4-5 indicate that hedgers do appear to increase their demand for futures contracts with decreases in this fixed cost, since the coefficient of $[C - r(t)s(t)]$ is significantly less than zero.

TABLE 4-5
RESULTS FROM ESTIMATION BY TWO STAGE LEAST SQUARES

A. Demand Equation

$R^2 = 0.0896$

Variable	Intercept	$[E[b(\tau)]-b(t)]$	$[C-r(t)a(t)]$	$[A_1(t)-A_2(t)]$
estimated coefficient:	442.679	21265.4	-1545.15	0.1306
t:	(3.598)	(4.312))	(-6.869)	(4.656)

B. Supply Equation

$R^2 = 0.2370$

Variable	Intercept	$[p(t)-E[p(\tau)]]$	$A_0 t$
estimated coefficient	858.004	5142.280	-0.068
t:	(19.573)	(4.921)	(-11.072)

The one variable in the demand equation for which we cannot reject the null hypothesis is the variable $[A_1(t) - A_2(t)]$. While the alternate hypothesis states that the coefficient of $[A_1(t) - A_2(t)]$ should be negative, the estimated coefficient is significantly greater than zero. It would appear that hedgers actually buy fewer futures contracts when the net amount of assets which could be hedged by buying futures contracts increases. Several interpretations of this result are possible. The first is that the model is incorrect, and hedgers do not respond to an increase in the net amount of assets which can be hedged using a long futures position by buying more futures contracts. While possible, such behavior does not seem plausible.

A second interpretation concerns the implications of an increase in the net amount of assets which could be hedged by long positions (that is, a decrease in $[A_1(t) - A_2(t)]$). It is possible that firms react to a decline in the expected spot price in one of two ways. The first is to hedge their assets by selling futures contracts. The second is to decrease or defer their holdings of these assets, hoping to buy them later at reduced prices, thereby decreasing the $[A_1(t) - A_2(t)]$ variable. According to the model, the decline in $[A_1(t) - A_2(t)]$ increases the potential for long hedging, but the reduction in $[A_1(t) - A_2(t)]$ might actually have been in response to circumstances under which we would have predicted a decrease in long hedging . . . an increase in $E[e(\tau)]$ (a decrease in $E[b(\tau)]$). Therefore, a decrease in $[A_1(t)-A_2(t)]$

might be the result of factors which would otherwise cause more short hedging rather than causing more long hedging. The net effect on long hedging of these two offsetting forces would be difficult to predict. It may be that the size of $[A_1(t) - A_2(t)]$ is more strongly affected by firms which reduce their asset holdings when they expect spot prices to fall, rather than taking a short hedge. As a result, the variable may increase at the same time as more net long hedging occurs.

A third interpretation of this result is that hedgers do respond to changes in assets as predicted, but that the $[A_1(t) - A_2(t)]$ variable does not provide a good measure of the assets which can be hedged by selling or buying GNMA future contracts. For example, it is possible that the variable may not represent firms' true changes in hedging needs because it is only measured on a monthly basis, rather than more frequently.

Of the three interpretations, the third seems to be the most likely. The transactions are observed daily, and increases or decreases of daily demand by hedgers should be a result of day-to-day changes in hedgers' needs and asset positions. Changes in needs and positions which are only measured monthly would not accurately reflect these day-to-day changes. Unfortunately, data for the sample were only available on a monthly basis, making it difficult to improve on the measurement of the $[A_1(t) - A_2(t)]$ variable at this time.

Based on the estimates for the supply equation in Table 4-5, the null hypothesis concerning the $[p(t) - E[p(t)]]$ coefficient can be rejected. If speculators are net short, they increase their supply of futures contracts with an increase in expected profit, measured as the current price minus the expected price. The supply function is upward-sloping with respect to price.

On the other hand, we cannot reject the null hypothesis concerning the coefficient of $A_0(t)$. Rather than being positive, the estimate for this coefficient is significantly negative, indicating that speculators decrease their selling activity when their asset bases increase. The alternate joint hypothesis from the model is that if speculators exhibit decreasing absolute risk aversion, and if they are net short, they will increase their net supply of futures contracts when initial wealth increases. We could draw one of several conclusions from the fact that the 2SLS estimate does not support this hypothesis. First, it is possible that the null hypothesis is correct; either speculators do not exhibit negative absolute risk aversion or they are not net short.

Another conclusion which could be reached is that the alternate hypothesis is actually correct, but that the magnitude of the effect of changes in initial wealth is larger for long hedgers than for short hedgers. We recall that the supply function is derived from the aggregate supply minus the aggregate demand for contracts by speculators. An increase initial wealth will increase speculators' demand for contracts as well as their supply of contracts. Speculators who buy contracts are those who expect the futures price to rise, yielding some level of expected profit. If these speculators exhibit decreasing absolute risk aversion, they will increase their demand for futures contracts with an increase in initial wealth, just as short speculators will increase their supply of contracts with increase in initial wealth. Now, if an increase in aggregate wealth, $A_0(t)$, affects long speculators more than short speculators, there could be a negative relationship between $A_0(t)$ and contracts supplied even when the alternate hypothesis is true. For example, a one percent increase in $A_0(t)$ might result in a two percent increase in speculators' demand and a one and a half percent increase in speculators' supply. The result would be a net decrease in speculators' supply of contracts when $A_0(t)$ increases, even though speculators are net short and exhibit decreasing absolute risk aversion. An increase in $A_0(t)$ could have this differential effect on speculating suppliers and demanders if the variable systematically reflects a larger proportionate change in demanders' initial wealth positions than that of suppliers, or if the two groups simply respond differently to equal proportionate increases in their respective initial wealth bases.

A final conclusion which might be reached concerning the $A_0(t)$ variable is that the null hypothesis should be rejected, but that $A_0(t)$ does not provide an accurate measure of the aggregate initial wealth of all speculators. The measurement of $A_0(t)$ as the average daily market value of government securities held by dealers for each week was done on the assumption that the majority of the speculating is done by dealers, and that weekly measurement of their government security holdings sufficiently captures any changes in their initial asset bases. It is possible that one or both of these assumptions are inappropriate, and that $A_0(t)$ should be measured in another fashion.

Of the three possible conclusions discussed, the second and third are the most likely, indicating a need for another means of measuring $A_0(t)$. However, a better measurement technique may not be possible without specifically identifying all traders who hold speculative positions, and inquiring about their long and short positions and assets. Rather than wait for this unlikely occurrence, we must accept the null hypothesis concerning the effect of $A_0(t)$ on the number of contracts supplied.

In summary, the 2SLS estimates would lead us to reject the null hypothesis concerning all variables except $[A_1(t) - A_2(t)]$ and $A_0(t)$. The estimated coefficients differ in magnitude from the OLS estimates, and the sign of the coefficient for the basis, $[E(b(t)) - b(t)]$ is actually reversed from the OLS estimate. These differences may be accounted for by the fact that variables containing price are correlated with the error terms of both the supply and demand equations, making OLS estimates inconsistent. Since the 2SLS process removes this inconsistency, it is not surprising to see changes in the resulting estimates. The only problem that remains is due to the fact that the price hits upper and lower limits during some of the days in the sample, which will be discussed in Chapter V.

CHAPTER V
DISEQUILIBRIUM ESTIMATION TECHNIQUES

This chapter is a discussion of the correct estimation techniques to be used for the interest rate futures market, and the usefulness of these techniques is analyzed in the context of our sample.

We first recall our system of equations of supply and demand for interest rates futures:

$$D_t = \beta_{11} + \beta_{12}\{E[b(T)] - b(t)\} + \beta_{13}[C - r(t)a(t)] + \beta_{14}[A_1(t) - A_2(t)] + u_t \qquad (1)$$

$$S_t = \beta_{21} + \beta_{22}\{p(t) - E[p(T)]\} + \beta_{23}A_0(t) + v_t \qquad (2)$$

$$Q_t = \min(D_t, S_t) \qquad (3)$$

$$|\Delta p| = \min(|p^*(t) - p(t-1)|, L) \qquad (4)$$

There are a total of 740 observations, which are separated into three sets; ψ_1, containing 732 observations which come from an equilibrium market; ψ_2, containing 3, observations from the supply side, where $\Delta p = +L$, and ψ_3, containing 5 observations from the demand side, where $\Delta p = -L$. For the set of observations in ψ_1, (1) and (2) form a set of simultaneous equations where $Q_t = D_t = S_t$, and Q_t and $p(t)$ are endogenous. For the observations in ψ_2 and ψ_3 however, $p(t)$ is an exogenous variable.

The parameters of the structural model should be estimated using the maximum likelihood technique. The correct likelihood function, which includes the three different regimes must first be developed. Let us denote by π_{st} the probability that the upper price limit has been reached, and that the observations come from the supply schedule. Then the conditional density of Q_t in the set ψ_2 is

$$f_s(Q_t) = \int_{Q_t}^{\infty} g_2(D_t,Q_t)dD_t/\pi_{st} \qquad (5)$$

where g_2 is the joint density of (D_t,Q_t) derived from the joint density of (u_t,v_t) from (1) and (2) where p(t) = p(t-1) + L and is an exogenous variable.

Similarly, we define π_{dt} as the probability of reaching the lower price limit, when the observations come from the demand side. The conditional density of Q_t in the set ψ_3 is then

$$f_d(Q_t) = \int_{Q_t}^{\infty} g_1(Q_t,S_t)dS_t/\pi_{dt} \quad . \qquad (6)$$

Here, g_1 is the joint density of (Q_t,S_t), derived from the joint density of (u_t,v_t) from (1) and (2) where p(t) = p(t-1) - L, and is exogenous.

Finally, the conditional joint density of Q_t and p(t) in the set ψ_1 (where both Q_t and p(t) are endogenous), is $f(Q_t, p(t))$, where f is derived from the joint density of (u_t,v_t) from (1) and (2) for $t\varepsilon\psi_1$.

The correct unconditional likelihood function to be maximized is therefore

$$L = \prod_{\psi_1} f(Q_t, p(t)) \cdot \prod_{\psi_2} \int_{Q_t}^{\infty} g_2(D_t, Q_t)\,dD_t \cdot \prod_{\psi_3} \int_{Q_t}^{\infty} g_1(Q_t, S_t)\,dS_t. \quad (7)$$

This is the product of f, f_s and f_d, all multiplied by the probability of obtaining the observed sample separation. Because the Jacobian of the transformation for (u_t, v_t) to $(Q_t, p(t))$ is $(\beta_{12} + \beta_{22})$ (since $b(t) = p(t) - a(t)$), then

$$f(Q_t, p(t)) = (\beta_{12} + \beta_{22}) f(u_t, v_t). \quad (8)$$

Based on the previous discussion, since $\sigma_p > [p(t) - E[p(\tau)]]^2$ then β_{22} is expected to be positive. Since β_{12} is also expected to be positive, $(\beta_{12} + \beta_{22})$ is expected to be positive. Similarly, the Jacobian of the transformation from (u_t, v_t) to (D_t, S_t) is unity, and

$$\int_{Q_t}^{\infty} g_2(D_t, Q_t)\,dD_t = \int_{a_t}^{\infty} f(u_t, v_t)\,du_t \quad (9)$$

where $a_t = Q_t - \beta_{21} - \beta_{22}(p(t) - E[p(\tau)]) - \beta_{23} A_0(t)$

Similarly,

$$\int_{Q_t}^{\infty} g_1(Q_t, S_t)\,dS_t = \int_{b_t}^{\infty} f(u_t, v_t)\,dv_t \quad (10)$$

where $b_t = Q_t - \beta_{11} - \beta_{12}\{E[b(\ )] - [illegible]$

$- \beta_{13}[\ - r(t)s(\ )] - \beta_{14}[\lambda_1(t) - \lambda_2(t)]$

Using (8), (9), and (10), (7) can be written as

$$L = \prod_{\psi_1} f(u_t,v_t)(\beta_{12}+\beta_{22}) \cdot \prod_{\psi_2} \int_{a_t}^{\infty} f(u_t,v_t)du_t \cdot \prod_{\psi_3} \int_{b_t}^{\infty} f(u_t,v_t)dv_t \qquad (11)$$

Assuming u_t and v_t are independently normally distributed, (11) can be written more explicitly as

$$L = (\beta_{12}+\beta_{22})^{732} \cdot \prod_{\psi_1+\psi_2} f(u_t) \cdot \prod_{\psi_1+\psi_3} f(v_t) \cdot \prod_{\psi_2} \int_{a_t}^{\infty} f(v_t)du_t \cdot \prod_{\psi_3} \int_{b_t}^{\infty} f(u_t)dv_t \qquad (12)$$

where $f(x) = 1/\sqrt{2\pi\sigma^2} \cdot \exp\{-\frac{1}{2}(x-\mu)^2/\sigma^2\}$, $x \sim N(\mu,\sigma^2)$.

The maximum likelihood estimates of β, and σ_u, σ_v are found by maximizing the log of the likelihood function in (12) through an iterative method such as the method of Newton or the one proposed by Berndt et. al (1974). However these procedures require initial consistent estimators in order to obtain second round estimators which are asymptotically efficient. The two stage procedure to obtain consistent initial estimators will now be described.*

The system of simultaneous equations, (1) and (2) for the set ψ_1 are first considered. In ψ_1, $p(t)$ is an endogenous variable. The reduced form for $p(t)$ can be expressed as

$$p(t) = \Pi X_t + \omega_t, \quad \text{where } \omega_t \sim \text{IID}(0,\sigma_\omega^2) \qquad (13)$$

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* This procedure is based on Maddala (1980).

Here X is a (3x1) vector of all the exogenous variables in the system and Π is a (1x3) vector of the reduced form coefficients. In order to estimate the parameters in Γ, the tobit method is applied to the following model

$$p(t) = \begin{cases} p(t-1) - L & \text{if } p(t) \le p(t-1) - L \\ \Pi X_t + \omega_t & \text{if } p(t-1) - L < p(t) < p(t-1) + L \\ p(t-1) + L & \text{if } p(t) \ge p(t-1) + L \end{cases} \qquad (14)$$

From this model, the likelihood function to be maximized is

$$L' = \prod_{\psi_1} f(\omega_t) \cdot \prod_{\psi_2} F_{2t}(\Pi X_t, \sigma^2) \cdot \prod_{\psi_3} [1 - F_{3t}(\Pi X_t, \sigma^2)] \quad (15)$$

where

$$F_{2t}(\Pi X_t, \sigma^2) = \int_{\infty}^{\Pi X_t - (p(t-1)+L)} \frac{1}{\sqrt{\pi\sigma^2}} \exp[-\tfrac{1}{2}\frac{(\lambda)^2}{\sigma^2}]\, d\lambda$$

and

$$F_{3t}(\Pi X_t, \sigma^2) = \int_{\infty}^{\Pi X_t - (p(t-1)-L)} \frac{1}{\sqrt{\pi\sigma^2}} \exp[-\tfrac{1}{2}\frac{(\lambda)^2}{\sigma^2}]\, d\lambda$$

The maximum likelihood estimation of (15) involves the use of a two-limit tobit procedure. The two-limit tobit model is based on extensions of the original tobit model, as listed above. The computation process is more difficult than the one used for the two-limit probit model, described in Rosett and Nelson (1975). While the standard tobit models are relatively [illegible] compute, with statistical programs available to apply to such application, the two-limit problem is not commonly encountered. An example of its application is found in Fishe and Lahiri (1981).

Tobit estimates are then used to get estimates, p(t), for p(t) for the equilibrium observations where $t \epsilon \psi_1$. This is done by first writing the expression for the expected value of p(t), given that the observation comes from the set of equilibrium points.

$$E[p(t) | t \epsilon \psi_1] = \beta X_t + E(u_t | \frac{p(t-1) - L - \beta X_t}{\sigma_u} < \frac{u_t}{\sigma_u} < \frac{p(t-1) + L - \beta X_t}{\sigma_u})$$

which can be written as

$$= \beta X_t + \sigma_u E(u_t | \frac{p(t-1) - L - \beta X_t}{\sigma_u} < \frac{u_t}{\sigma_u} < \frac{p(t-1) + L - \beta X_t}{\sigma_u})$$

or $$E[p(t) | t \epsilon \psi_1) = \beta X_t + \sigma_u \frac{f(Z_{1t}) - f(Z_{2t})}{F(Z_{1t}) - F(Z_{2t})} \qquad (16)$$

where $$Z_{1t} = \frac{p(t-1) - L - \beta X_t}{\sigma_u}$$

and $$Z_{2t} = \frac{p(t-1) + L - \beta X_t}{\sigma_u}$$

and f(·) and F(·) are defined as follows.

$$f(Z_{it}) = 1/\sqrt{2\pi}\ \exp\left(-\tfrac{1}{2} Z_{it}^2\right) \qquad i = 1,2 \qquad (17)$$

and

$$F(Z_{it}) = \int_{-\infty}^{Z_{it}} 1/\sqrt{2\pi}\ \exp\left(-\tfrac{1}{2}(\ )^2\right) d\ \qquad i = 1,2 \qquad (18)$$

The estimates, $\hat{p}(t)$, for p(t) are based on equation (16), using the estimates $\hat{\sigma}_\omega$ and $\hat{\Pi}$ derived from the maximization of equation (15).

The second step on this two stage procedure is to use the values of $\hat{p}(t)$ in the supply and demand equations, (1) and (2) for all the equilibrium observations, along with the appropriate value for the exogenous p(t) variable for the observations in ψ_2 and ψ_3. Before the estimation itself, the equations in each regime must be adjusted for the fact that the residuals no longer have zero means in any regime considered. We first examine the means of the residuals in the supply and demand equations for $t \varepsilon \psi_1$.

$$E(u_t | t \varepsilon \psi_1) = E\left(u_t \middle| \frac{p(t-1) - L - \Pi X_t}{\sigma_\omega} < \frac{\omega_t}{\sigma_\omega} < \frac{p(t-1) + L - \Pi X_t}{\sigma_\omega}\right)$$

$$= \frac{\sigma_{u\omega}}{\sigma_\omega} W_{1t} \qquad (19)$$

For equation (19) and the ones which follow, the following definitions are used:

$$W_{1t} = \frac{f(Z_{1t}) - f(Z_{3t})}{F(Z_{2t}) - F(Z_{1t})} \tag{20}$$

$$W_{2t} = \frac{f(Z_{2t})}{1 - F(Z_{2t})} \tag{21}$$

$$W_{3t} = \frac{-f(Z_{1t})}{F(Z_{1t})} \tag{22}$$

and $\sigma_{i\omega} = \text{cov}(i\ ,\ \omega)\ ,\qquad i = u,v.$

Just as we did for equation (19), we now examine the residuals for the supply equation for $t\varepsilon\psi_1$.

$$E(v_t | t\varepsilon\psi_1) = E\left(v_t \middle| \frac{p(t-1) - L - \Pi X_t}{\sigma_\omega} < \frac{\omega_t}{\sigma_\omega} < \frac{p(t-1) + L - \Pi X_t}{\sigma_\omega}\right)$$

$$= \frac{\sigma_{v\omega}}{\sigma_\omega} W_{1t} \tag{23}$$

For the subset ψ_2, $p(t) = p(t-1) + L$ and (2) is observed. Therefore, for these observations,

$$E(u_t | t\epsilon\psi_2) = E(u_t | \frac{\omega_t}{\sigma_\omega} > \frac{p(t-1) + L - \Pi X_t}{\sigma_\omega})$$

$$= \frac{\sigma_{u\omega}}{\sigma_\omega} W_{2t} \qquad (24)$$

Finally, for the subset ψ_3, $p(t) = p(t-1) - L$. These observations come from equation (29), where we have

$$E(v_t | t\epsilon\psi_3) = E(v_t | \frac{\omega_t}{\sigma_\omega} < \frac{p(t-1) - L - \Pi X_t}{\sigma_\omega})$$

$$= \frac{\sigma_{v\omega}}{\sigma_\omega} W_{3t} \qquad (25)$$

Combining all the information contained in equations (19), (23), (24) and (25) results in the following system of equations for the second stage estimation:

Demand functions

$$Q_t = \beta_{11} + \beta_{12}\{E[b(\tau)] - [p(t) - s(t)]\} + \beta_{13}[C - r(t)s(t)] + \beta_{14}[A_1(t) - A_2(t)] + \frac{\sigma_{u\omega}}{\sigma_\omega}W_{1t} + \eta_{1t},\ t\epsilon\psi_1 \qquad (26)$$

$$Q_t = \beta_{11} + \beta_{12}\{E[b(\tau)] - [p(t-1) - L - s(t)]\} + \beta_{13}[C - r(t)s(t)] + \beta_{14}[A_1(t) - A_2(t)] + \frac{\sigma_{u\omega}}{\sigma_\omega}W_{3t} + \eta_{1t},\ t\epsilon\psi_3. \qquad (27)$$

Supply functions

$$Q_t = \beta_{21} + \beta_{22}\{p(t) - E[p(\tau)]\} + \beta_{23}A_o(t) + \frac{\sigma_{v\omega}}{\sigma_\omega}W_{1t}$$

$$+ \eta_{2t},\ t \in \omega_1. \tag{28}$$

$$Q_t = \beta_{21} + \beta_{22}\{p(t-1) + L - E[p(\tau)]\} + \beta_{23}A_o(t)$$

$$+ \frac{\sigma_{v\omega}}{\sigma_\omega}W_{2t} + \eta_{2t},\ t \in \upsilon_2, \tag{29}$$

These new residuals, η_{1t} and η_{2t} all have zero means. The second stage estimation of equations (26) through (29) proceeds after substituting $\hat{p}(t)$ for $p(t)$ and $\hat{W}_{1t}$ for W_{1t} in equations (26) and (28). Similarly, $\hat{W}_{3t}$ and $\hat{W}_{2t}$ are substituted into equations (27) and (29) respectively for the values, W_{3t} and W_{2t}. These estimates, $\hat{W}_{it}$, are obtained using the values for $\hat{p}(t)$ and $\hat{\Pi}$ in equations (20) through (22). Ordinary least squares techniques are then used to estimate the equations (26) and (27) together, and (28) and (29) together.

The resulting estimates, "tobit two-stage least squares" estimates, are consistent. They may be used as initial consistent estimators in the computation of the maximum likelihood estimates in (13).

As we have stated, the procedure just outlined is the correct estimation technique for markets which have upper and lower limits on the daily price movement. If many of the observations in the sample are disequilibrium observations with the price at the upper or lower limit, then estimates of the parameters from this procedure may be very different from 2SLS estimates. If one half, or even one third of our 740 observations had prices at one of the two limits, the 2SLS estimation would be done on a sample in which a large portion of the observations did not have price as an endogenous variable as assumed when using 2SLS. Furthermore, knowing whether the quantity transacted represents only the supply or the demand schedule provides added information which is being ignored by using 2SLS, and will change the estimates to account for the fact that the other schedule (demand or supply) is not also represented.

If the number of disequilibrium observations is small in proportion to the size of the sample, however, it is very likely that the maximum likelihood estimates from the two-limit tobit initial estimates will be very close to the 2SLS estimates. Preliminary calculations indicate that this would be the case for our sample. Hence, the elaborate and costly procedure of estimating the model using a maximum likelihood technique was not done. For our sample, disequilibrium observations accounted for just over one percent of all observations, a very small proportion. In order to see if the disequilibrium observations had a large influence on the estimate, we applied 2SLS to the sample with the disequilibrium points omitted. The results are listed in Table 5-1. Comparing these estimates with those in Table 4-5, we see that the 2SLS estimates change very little with the exclusion of the disequilibrium observations. Similar evidence was found in an examination of the reduced form estimates. For the first step of the 2SLS estimation, the price is regressed on all of the exogenous variables in the set of supply and demand equations using OLS to obtain the variable $\hat{p}(t)$ to replace p(t) for the second stage. For the maximum likelihood technique, a similar procedure is done for the first stage of the two-stage tobit technique, where $\hat{p}(t)$ is found from the tobit estimates when p(t) is regressed on the exogenous variables. The $\hat{p}(t)$ from the tobit estimation is used to replace p(t) in the second stage, just as the OLS $\hat{p}(t)$ is used in the second stage of the 2SLS estimation. If the $\hat{p}(t)$ variable obtained from the OLS estimation is not greatly different from that of the tobit estimation, the second stage of the 2SLS will contain variables which are similar to those in (26) and (28), where $\hat{p}(t)$ is used instead of p(t) for the second stage of the tobit two stage estimation. Therefore, we would expect the resulting consistent tobit estimates to be close to the 2SLS estimates in these circumstances. The results of the OLS estimation on the reduced form are contained in Table 5-2. The reduced form was then estimated using a standard one-limit tobit model based on Fair (1977), in order to have some indication of the effect of adjusting for the disequilibrium points. The three observations in which the price reached the upper limit were omitted, resulting in a sample with prices reaching only one

TABLE 5-1
RESULTS FROM ESTIMATION BY TWO STAGE LEAST SQUARES
WHEN DISEQUILIBRIUM POINTS ARE OMITTED

A. Demand Equation

$R^2 = 0.0896$

Variable	Intercept	$\{E[b(\tau)]-b(t)\}$	$[C-r(t)s(t)]$	$[A_1(t)-A_2(t)]$
estimated coefficient:	418.502	23084.9	-1673.71	0.1444
t:	(3.172)	(4.087)	(-6.310)	(4.502)

B. Supply Equation

$R^2 = 0.2460$

Variable	Intercept	$\{p(t)-E[p(\tau)]\}$	A_0t
estimated coefficient.	841.256	5175.18	-0.064
t:	(18.514)	(5.202)	(-9.713)

limit. The results of the tobit estimation are listed in Table 5-3. Comparing these results with those in Table 5-2, we can see that the tobit technique has caused very little change in the reduced form estimates.

From the foregoing evidence, we must conclude that the benefit from undertaking the elaborate computations of the two-stage tobit technique must be very small for our sample. While the procedure outlined will be useful in problems with a larger proportion of limit observations--most futures markets, during days when the spot asset exhibits large price changes--it will not lead us to results much different from the 2SLS estimates for our particular sample.

Therefore, we would expect to reach the same interpretations and conclusions based on the two-stage tobit estimates as those based on the 2SLS estimates in Chapter IV.

TABLE 5-2
RESULTS FROM ESTIMATION BY ORDINARY LEAST SQUARES OF THE REDUCED FORM EQUATION

Dependent Variable: $p(t)$

Variable	Intercept	$[C - r(t)s(t)]$	$[A_1(t) - A_2(t)]$	$A_0(t)$	$p(t-1)-L$
estimated coefficient:	1.279	0.1094	0.000011	0.000005	0.0070
standard deviation:	(0.382)	(0.0393)	(0.000006)	(0.000005)	(0.0043)

TABLE 5-3
RESULTS FROM ESTIMATION BY THE TOBIT TECHNIQUE
OF THE REDUCED FROM EQUATION

Dependant variable: $p(t)$

Variable	Intercept	$[C - r(t)s(t)]$	$[A_1(t) - A_2(t)]$	$A_o(t)$	$p(t-1)$
estimated coefficient	1.191	0.0950	0.000011	0.000005	0.0061
standard deviation:	(0.378)	(0.0391)	(0.000006)	(0.000005)	(0.0042)

CHAPTER VI
SUMMARY AND CONCLUSIONS

A model of demand for and supply of interest rate futures contracts is developed in Chapter III. The model is based on previous work by Stein (1979), pertaining to commodity futures contracts. The demand function is derived first, based upon the needs and actions of hedgers, assumed to be net long. After deriving an expression for the optimal fraction of assets which should be hedged, the factors which affect the demand for interest rate futures are identified. Because of the manner in which the factors affect the optimal fraction of assets to be hedged, predictions can be made concerning the direction of the effect on demand resulting from an increase in each factor. According to the model, an increase in the expected return from the hedged position--the expected increase in the basis--will result in an increase in the demand for contracts. An increase in the net opportunity cost of not holding GNMA certificates--the coupon payment minus the opportunity cost of the funds employed--will decrease the number of contracts demanded by hedgers. Finally, a net increase in the amount of assets held (and needs for such assets) which could be hedged by selling interest rate futures, will decrease the demand for contracts.

The supply function is derived by examining the actions of speculators, who are assumed to maximize expected utility. A general form of a utility function for a speculator is written as a Taylor series expansion, using only the first three terms. The expected utility is written out, and the number of futures contracts which maximize expected utility is derived. Factors affecting the supply of contracts are identified from this expression. The expression also provides the information necessary to make predictions concerning the direction of effect on the supply of interest rate futures contracts resulting from an increase in each factor. Based on the model and the behavior of futures prices, an increase in the expected profit from selling a contract--the expected decrease in the futures price--will increase the supply of contracts. Assuming decreasing absolute risk aversion, an increase in the initial wealth of speculators will increase the supply of contracts.

The sample description and the definition of the variables are presented in Chapter IV, which also contains the results of estimation by OLS and 2SLS. The results from the OLS estimation would indicate that both supply and demand for futures contracts increase when the futures price increases. The two variables measuring assets, $A_0(t)$ and $[A_1(t) - A_2(t)]$, do not appear to affect supply and demand as predicted. When estimation is done by 2SLS, the correlation of the price variable and the error term is removed. The resulting estimates indicate that supply increases when the futures price increases, while demand decreases when the futures price increases. The 2SLS estimates of the coefficients for the two variables measuring assets have the same signs as the OLS estimates. The most likely reason for these results is that the model is accurate, but that the measurement of $A_0(t)$ and $[A_1(t) - A_2(t)]$ is not precise enough to capture the relevant changes in these factors. Unfortunately, a different way to measure these two variables does not appear to be plausible at this time.

Chapter V is a discussion of the correct estimation techniques for markets such as the interest rate futures market, in which the price is subject to upper and lower limits. Because the sample being studied has relatively few disequilibrium points, it is very likely that the application of these elaborate techniques will yield estimates which are extremely close to the 2SLS estimates. Preliminary testing of the sample indicates that this is the case. Therefore, the interpretation of the tobit results is most likely to be the same as that from the 2SLS results presented in Chapter IV.

While it is likely that the tobit technique would not affect our conclusions drawn from the 2SLS estimates, it is important to remember that the tobit technique is indeed the correct one for such samples. Futures markets often have a much larger proportion of disequilibrium days. When testing this model for later time periods in the GNMA market as well as other markets, it is likely that the tobit technique will provide a great deal of information not possible from the 2SLS technique.

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BIOGRAPHICAL SKETCH

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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March, 1981

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